國 立 清 華 大 學 命 題 紙

99 學年度 工業工程與工程管理學系甲、乙、丙組 碩士班入學考試

科目 統計學 科目代碼 1401、1501、1601 共 3 頁第 1 頁 *請在【答案卷卡】內作答

1. (15 pts.) If X, Y, and Z are positive valued and all have means of 6 and standard deviations of 3, estimate the probability that the average of X, Y and Z is less than 12 by Chebyshev's inequality.

   (a) (5 pts.) If X, Y, and Z are independent.

   (b) (10 pts.) If X, Y and Z all have correlation coefficients of 0.3.

2. (10 pts.) Suppose the lifetime of an electronic tube has an exponential distribution with a mean time of μ hours. Find the expected lifetime of a tube given that the tube has lasted at least T hours.

3. (15 pts.) Consider the hypothesis testing $H_0 : p = 1/4$ versus $H_1 : p > 1/4$, where p is the probability of getting a head of a coin. Suppose the coin is flipped 100 times and 30 heads are obtained. Perform this test at 5% of significance level.

   (a) (5 pts.) Find P-value of the test and draw conclusion.

   (b) (10 pts.) Find the type II error of the test at $p = 0.4$.

4. (10 pts.) A quality control engineer with Systems Planning Corporation would like to estimate the proportion of defects being produced on a production line to within ± 0.05 with 95% confidence. In a preliminary sample of 25 items the engineer found 4 defectives. How many samples would be needed for achieving her goal?

5. (8 pts.) Given the probabilities of some events as follows: $\mathrm{P}(C|B) = 0.94$, $\mathrm{P}(C|B^{\mathrm{c}}) = 0.3$, $\mathrm{P}(A|B \cap C) = 0.1$, $\mathrm{P}(A|B^{\mathrm{c}} \cap C) = 0.05$, $\mathrm{P}(A|B \cap C^{\mathrm{c}}) = 0.35$, $\mathrm{P}(A|B^{\mathrm{c}} \cap C^{\mathrm{c}}) = 0.20$. Compute

   (a) (4 pts.) $\mathrm{P}(A|B)$

   (b) (4 pts.) $\mathrm{P}(A|B^{\mathrm{c}})$

6. (10 pts.) Continue the previous question (Question 5). Let A be the event that any student who passes an English test, B be the event that any student who has an English tutor, and C be the event that any student who is male. Comparing and explaining in your word about the two conditional probabilities listed in each subproblems. You also need to comment whether these comparisons of two probabilities match your intuition. Why or why not.

(a) (3 pts.) $P(A|B \cap C)$ and $P(A|B^c \cap C)$

(b) (3 pts.) Compare $P(A|B \cap C^c)$ and $P(A|B^c \cap C^c)$

(c) (4 pts.) Compare $P(A|B)$ and $P(A|B^c)$

7. (20 pts.) True or False

(a) _____ A statistic is a random variable.

(b) _____ mean squared error of an estimator $\hat{\Theta}$ is the sum of the variance and the bias of the estimator.

(c) _____ When we choose from a set of possible point estimators, a reasonable approach is to choose the estimator with the largest mean squared error.

(d) _____ A sample mean $\overline{X}_n = \sum_{i=1}^{n} X_i/n$ follows a normal distribution for any n if $X_1, \ldots, X_n$ are i.i.d. Normal random variables.

(e) _____ Suppose that $X_1, X_2, .., X_n$ is a random sample from a Bernoulli distribution with mean p. Then $\sum_{i=1}^{n} X_i$ follows a geometric distribution.

(f) Suppose that X_1, X_2, ..., X_n is a random sample of size n $(n > 3)$ taken from a normal distribution with unknown mean μ. Let $\overline{X}$ be the sample mean of X_1, X_2, ..., and X_n. The goal is to estimate μ.

i. _____ In terms of the bias, $\overline{X}$ is a better estimator than X_1.

ii. _____ In terms of the variance, $\overline{X}$ is a better estimator than X_1.

iii. _____ In terms of the mean-squared-error, $\overline{X}$ is a better estimator than X_1.

(g) _____ Consider a one-sided test for the unknown parameter θ:
H_0: $\theta = 1$, H_1: $\theta > 1$. Suppose that the sampling distribution of $\hat{\theta}$ is normal. If the $\hat{\theta}$, the estimate of θ, is 0.9, we will always fail to reject H_0 if the significance level α is less than 0.5.

(h) _____ Suppose that voters, choosing between a Republican and a Democratic candidate, give the Republican $p \times 100\%$ of

the votes. We take a random sample of all voters. Consider the statistical hypotheses:
H_0: $p \leq 0.5$, H_1: $p > 0.5$. Suppose that we take a sample of size n and make a decision that we reject the null hypothesis. That means that the testing-hypothesis approach has helped us to see the true: "Republican will win".

8. (12 pts.) There are 12 plots ((a) to (l)) in Figure 1, including six probability density or mass functions (pdf or pmf) and their corresponding cumulative distribution functions (cdf).

   (a) List the six pdfs and pmf.

   (b) List the corresponding cdf for each pdf listed in the previous subproblem. and pmf

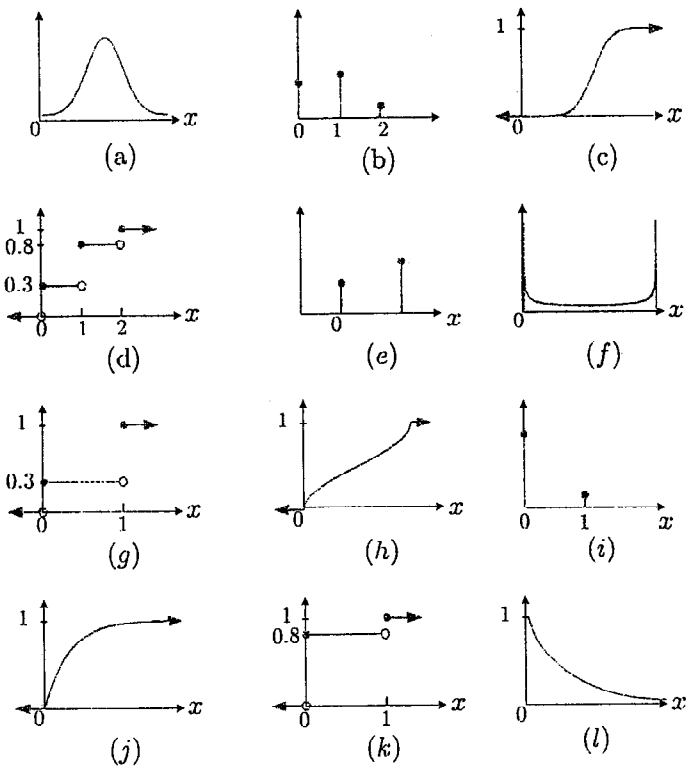

Figure 1: 12 plots