

國立清華大學命題紙

九十三學年度 微機電系統工程研究所 系(所) 甲 組碩士班入學考試

科目 電磁學 科號 2104 共 2 頁第 1 頁 \*請在試卷【答案卷】內作答

1. Determine true or false for the following nulls and give a brief explanation.

(a)  $\nabla(\nabla \times \vec{A}) = 0$ , where  $\vec{A}$  is any vector in space.

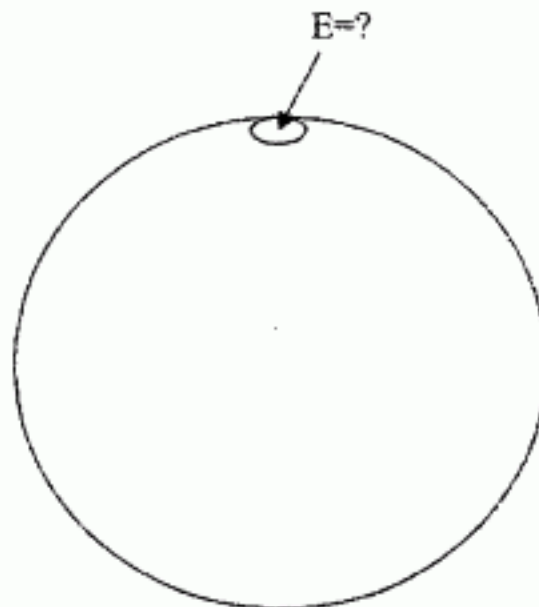
(b)  $\nabla \times (\nabla V) = 0$ , where  $V$  is any scalar in space.

(c)  $\nabla \times (\nabla \cdot \vec{A}) = 0$ , where  $\vec{A}$  is any vector in space.

(d)  $\nabla \cdot (\nabla V) = 0$ , where  $V$  is any scalar in space.

(e)  $\nabla \cdot (\nabla \times \vec{A}) = 0$ , where  $\vec{A}$  is any vector in space.

2. There is a hole on a charged ( $Q_0$ ) metal sphere shell. Please find the electric field in the center of the hole. Note that the hole is negligible in size.



3. Determine the resistance between two concentric spherical surfaces of radii  $R_1$  and  $R_2$  ( $R_1 < R_2$ ), assuming that a material of conductivity  $\sigma = \sigma_0(1 - k/R)$  fills the space between them. (Note: Laplace's equation for  $V$  does not apply here)

4. A toroidal core has a rectangular cross section defined by the surfaces  $R=2\text{cm}$  (radius),  $R=3\text{cm}$ ,  $z=4\text{cm}$ , and  $z=5\text{cm}$ . The core material has a relative permeability of 60. If the core is wound with a coil containing 10000 turns of wire, find its inductance.

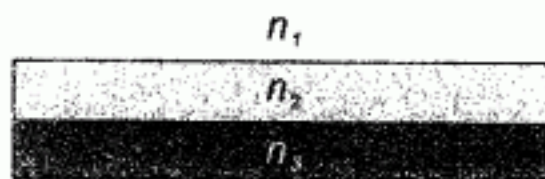
5. A kind of liquid solution at frequency  $\nu = 8 \times 10^8 \text{ Hz}$  has permittivity  $\epsilon = 80\epsilon_0$ , permeability  $\mu = \mu_0$ , and resistivity  $\rho = 0.2 \Omega \cdot \text{m}$ . What is the ratio of conduction current to displacement current?

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科目 電磁學 科號 2404 共 2 頁第 2 頁 \*請在試卷【答案卷】內作答

6. A lens made of glass with refractive index 1.6 has a focal length of 0.5 m in air. What is the focal length in water? The refractive index of water is 1.333.
7. What is the origin of rainbow? Consider the geometry of a parallel beam of sunlight striking a spherical rain drop, prove that there are actually two rainbows. Show your reasoning by drawing a picture.
8. Two equal sources radiate a wavelength  $\lambda$  and are separated a distance  $\lambda/4$ . There is a phase difference  $\delta_0 = \pi/2$  between the signals at source. If the intensity of each source is  $I_s$ , show that the intensity of the radiation pattern is given by  $I = 4I_s[\cos^2 \frac{\pi}{4}(1 + \sin \theta)]$ , where  $\theta$  is the angle of observation.
9. Light of wavelength  $\lambda$  in a medium of refractive index  $n_1$  is normally incident on a thin film of refractive index  $n_2$  and optical thickness  $\lambda/4$  which coats a plane substrate of refractive index  $n_3$ . Show that the film is a perfect anti-reflector (reflectivity is zero) if  $(n_2)^2 = n_1 n_3$ .



10. A step-index optical fiber shown below has a central core of index  $n_1$  surrounded by a cladding of index  $n_2$  where  $n_2 < n_1$ . From Snell's law, prove that the condition for the total internal reflection is:

$$\sin \theta_0 < \sqrt{n_1^2 - n_2^2} \text{ for light incident from air (refractive index = 1).}$$

