國　立　清　華　大　學　命　題　紙

98 學年度 統計學研究所 組碩士班入學考試

科目 統 計 學 科目代碼 0103 共 3 頁第 1 頁 *請在【答案卷卡】內作答

1~2 題是計算題，答題時，請將演算過程完整寫出。

1. Let X be a random variable generated from a distribution which has a probability mass function $p(x)$ that is positive on and only on the non-negative integers. Based on X, it is desired to test the simple hypothesis

   $H_0: p(x) = \frac{e^{-1}}{x!}, \ x = 0, 1, 2, \cdots,$

   against the alternative simple hypothesis

   $H_1: p(x) = (\frac{1}{2})^{x+1}, \ x = 0, 1, 2, \cdots.$

   Select a rejection region C so as to minimize $2\alpha + \beta$, where α and β are the respective probabilities of the type I and type II errors associated with C. (15%)

2. A random experiment that results in a success with probability θ, $0 < \theta < 1$, and a failure with probability $1 - \theta$ is called a Bernoulli experiment.

   (a) Suppose that a statistician observes 10 independent trials of a Bernoulli experiment. Let X denote the number of successes in these 10 trials. Based on X, can you find an unbiased estimator of $\frac{1}{\theta}$? Explain. (10%)

   (b) Instead of taking 10 observations, if the statistician had decided to take as many observations as needed to get the first success. Let Y denote the number of needed observations. Based on Y, find the maximum likelihood estimator (m.l.e) of $\frac{1}{\theta}$. Is the m.l.e. an unbiased estimator of $\frac{1}{\theta}$? Explain. (10%)

3~11 題是簡答題，答題時，只要寫出最後答案。

3. Let $\{-1, -4, 5, 3, -6\}$ be a random sample of size 5 from the uniform distribution over the interval $(-\theta, \theta)$, $0 < \theta < \infty$. Find the maximum likelihood estimate of θ. : _____(5%)

4. Let $X_1, X_2, \cdots, X_n$ be a random sample from the uniform distribution over the interval $(\theta-1, \theta+1)$, where $-\infty < \theta < \infty$. Find the minimum sufficient statistics for θ. : _____ (5%)

5. Review **problem 4**, find the UMVUE of θ. : _____(5%)

6. Let X be a random variable having a distribution with probability density $f(x;\theta) = \frac{1}{2}e^{-|x-\theta|}$, $-\infty < x < \infty$, where θ, $-\infty < \theta < \infty$, is an unknown location parameter. Based on X, construct a 90% confidence interval for θ. : _____(5%)

7. let X be a discrete type random variable which has a probability mass function $p(x)$ that is positive on and only on the natural numbers $\{1, 2, \cdots\}$. Assume that X has the following property: For any positive integers m and n, $\Pr\{X > m+n \mid X > m\} = \Pr\{X > n\}$. Find the function form of $p(x)$. :_____ (5%)

8. Suppose that the number of kilometers that a car can run before its battery wears out is exponentially distributed with an average value of 10,000 kilometers. A person decides to take a 5,000-kilometers trip. Suppose that the number of kilometers that the battery had been in use prior to the start of the trip is 3,000 kilometers. Compute the probability that he will be able to complete the trip without having to replace the car battery. : _____ (5%)

9. Repeat **problem 8** when the distribution is not exponential. Express your answer in terms of the cumulative distribution function $F(x)$ of the lifetime. : _____(5%)

10. To study the relationship between a dependent variable Y and 4 independent variables, a linear model, $Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$ was used and regression analysis was performed. The following information was obtained:

The sample size, n=21.

The sample variance of Y_i's, $S_y^2 = 800$.

The coefficient of determination, $R^2 = 0.6$.

Develop the ANOVA table for regression analysis. (15%)

Source of variation	Degrees of Freedom	Sum of squares	Mean squares	F statistics
Regression	(**A**=?)	(**D**=?)	(**G**=?)	(**I**=?)
Residual	(**B**=?)	(**E**=?)	(**H**=?)	*****
Total	(**C**=?)	(**F**=?)	*****	*****

11. A complete factorial experiment is an experiment in which the data in all possible combinations of the levels of the factors are gathered. In a complete two-way 3×4 factorial experiment with 5 replicates in each combination of the levels, a partial ANOVA Table is shown below:

Source of variation	Degrees of Freedom	Sum of squares	Mean squares	F statistics
Factor A	(**A**=?)	(**F**=?)	90	(**K**=?)
Factor B	(**B**=?)	90	(**H**=?)	(**L**=?)
Interaction	(**C**=?)	30	(**I**=?)	(**M**=?)
Error	(**D**=?)	(**G**=?)	(**J**=?)	*****
Total	(**E**=?)	1260	*****	*****

Fill in the missing values in the above ANOVA table. (15%)