國 立 清 華 大 學 命 題 紙

98學年度 計量財務金融學 系（所）乙組（財務工程組）硕士班入學考試

科目 微積分 科目代碼 4904 共 1 頁第 1 頁 *請在【答案卷卡】內作答

Total 100 points.

(1) (5 points each) Fill in Your Answers.

- $\lim_{n\to\infty} \sum_{i=1}^{n} \ln\left[\left(1+\frac{i}{n}\right)^{1/n}\right] =$ ______(a)______
- If $f(x) = \int_2^x \frac{dt}{\sqrt{1+t^4}}$, then $\left(f^{-1}\right)'(0) =$ ______(b)______
- Does $\sum_{k=1}^{\infty} \ln\left(1+\frac{1}{k^2}\right)$ converge or not? ______(c)______
- The plane tangent to the surface $z = x\cos y - y\,e^x$ at $(0, 0, 0)$ is ______(d)______
- The integral $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin and the point $(1, 1, 1)$ is evaluated as ______(e)______
- If $x = \int_0^y \frac{dt}{\sqrt{1+t^2}}$, then $\frac{1}{y}\frac{d^2y}{dx^2} =$ ______(f)______

(2) (10 points) Give an example that if two real-valued functions f and g satisfy $f'(x) \le g'(x)$ for each $x \in [a, b]$, then $f(x)$ may not be greater than or equal to $g(x)$. Please add a condition so that $f(x) \le g(x)$ for each $x \in [a, b]$.

(3) (a) (10 points) State the Fundamental Theorem of Calculus.

(b) (15 points) Let f be continuously differentiable in $\Re$ and $\int_0^{\infty} \frac{f(t)}{t}dt$ exist, calculate $\int_0^{\infty} \frac{f(\alpha x) - f(\beta x)}{x}dx$, for $\alpha > \beta$.

(4) (10 points) The plane $z = Ax + By + C$ is to be "fitted" to the following points (x_k, y_k, z_k) : $(0, 0, 0), (0, 1, 1), (1, 1, 1), (1, 0, -1)$. Find the values of A, B, and C that minimize $\sum_{k=1}^{4}(A\,x_k + B\,y_k + C - z_k)^2$.

(5) (10 points) Given the one-dimensional differential equation $\frac{dS(t)}{dt} = \alpha S(t) + \beta S(t)\frac{df(t)}{dt}$, where $t \ge 0$, α and β are constants, and $S(0) > 0$. Prove that $S(t)$ has a positive solution. [Hint: solve $S(t)$ by a logarithm transformation.]

(6) (5 points each) True or False.

(a) Let $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x-a) = 0$, then $\lim_{x\to 0} f(x+a)\,g(x) = 0$

(b) If f^2 is Riemann integrable on $[0, 1]$, then f is Riemann integrable.

(c) The series $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ converges.