

國立清華大學 100 學年度碩士班入學考試試題
系所班組別：計量財務金融學系碩士班 乙組(財務工程組)(0548)
考試科目(代碼)：微積分(4804) 共 1 頁，第 1 頁 *請在【答案卷、卡】作答

Calculus

1. (21 pts) Evaluate the following integrals:

(a) $\int_2^4 \frac{x^2+1}{x^2-x} dx.$

(b) $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1} dx dy.$

(c) $\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2} dx dy.$

2. (7 pts) Find $\frac{d}{dx} (x^{\sin x})$ at $x = \frac{\pi}{2}.$

3. (12 pts) If $x \sin x = \int_0^{x^2} f(y) dy$, where f is a continuous function, find

(a) $f(\pi^2).$

(b) $\lim_{x \rightarrow 0^+} f(x).$

4. (12 pts) Show that if $a_n > 0$ and $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is convergent.

5. (16 pts) The rate of change in sales S is modeled by

$$\frac{dS}{dt} = \frac{1}{2}(100 - S) + \frac{1}{4}t$$

where t is the time in days, and $S = 0$ when $t = 0$. Solve this differential equation for S as a function of t .

6. (16 pts) Find the absolute maximum and minimum values of

$$f(x, y) = e^{-x^2-y^2} (x^2 + 2y^2)$$

on the disk $\{(x, y) / x^2 + y^2 \leq 4\}.$

7. (16 pts) The production function for a company is given by

$$f(x, y) = 100x^{1/3}y^{2/3}$$

where x represents the units of labor and y represents the units of capital. Suppose that labor costs \$30 per unit, capital costs \$20 per unit, and management sets a production goal of 10,000 units. Find the numbers of units of labor and capital needed to meet the production goal while minimizing the cost.

「務必書寫計算過程，否則不予計分。」