

國立清華大學 100 學年度碩士班入學考試試題

系所班組別：資訊工程學系碩士班(0522)

考試科目（代碼）：基礎計算機科學(2201)

共 5 頁，第 1 頁 *請在【答案卷、卡】作答

1. (10%)

```
#include <stdio.h>
#include <string.h>
void swap(char *x, char *y)
/* swap the values pointed by x and y */
{
    char tmp;
    (a) ;
    (b) ;
    (c) ;
}
void permutation(char *start, char *end)
/* generate permutations recursively */
{
    char *cp;
    if ( (d) ) /* stopping criterion */
        printf( (e) ); /* display the current string */
    else
        for ( (f) ; (g) ; cp++)
        {
            swap( (h) );
            permutation( (i) ); /* recursive call */
            swap( (j) );
        }
}
int main(void)
{
    char str[] = {'1', '2', '3', '\0'};
    permutation(str, str+strlen(str)-1);
    return 0;
}
```

Complete the above C program to produce the permutations of 1, 2, 3 in the following order: 231, 321, 312, 132, 213, 123.

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共 5 頁，第 2 頁 *請在【答案卷、卡】作答

2. (6%) Consider a modification to merge sort in which n/k sub-lists of length k are sorted using quicksort and then merged using the standard merging mechanism, where n is the input size and $k \leq n$ is a given integer. Answer the following two questions in terms of n and k .
 - (a) (3%) What is the worst-case time complexity of the above modified merge sort?
 - (b) (3%) What is the average-case time complexity of the above modified merge sort?
3. (7%) A sequence of left and right parentheses is *well-formed* if the parentheses are properly nested. Give an efficient algorithm to determine whether a given sequence S of left and right parentheses is well-formed. Use $S = "((()())"$ as an example to explain your algorithm. Let $|S| = n$. What is the time complexity of your algorithm in terms of n ?
4. (7%) Let A be an array that stores n integers in $[1, 2^{\log \log n \times \log n}]$. Give an efficient algorithm to sort the integers in A . The time complexity of your algorithm must be better than $O(n \log n)$. What is the time complexity of your algorithm in terms of n ?
5. (a) (2%) Suppose the frequencies of the characters A, B, C, D, E, and F are 26, 9, 17, 11, 23, 14, respectively. Construct a Huffman tree for these characters.
(b) (5%) In general, let f_1, f_2, \dots, f_n denote the frequencies of n characters. Give an $O(n \log n)$ -time algorithm to construct a Huffman tree for these n characters.
6. (6%) Suppose we have n ranges $[a_1, b_1], [a_2, b_2], \dots, [a_n, b_n]$, where all a_i 's are negative, and all b_i 's are positive. We are asked to preprocess these ranges so that for any input value x , we can efficiently count the number of ranges containing x . Design an efficient representation of the ranges so that the desired counting can be done in $O(\log n)$ time. Your representation must take $O(n)$ space. Describe your representation and how to answer a query in $O(\log n)$ time in details.

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共 5 頁，第 3 頁 *請在【答案卷、卡】作答

7. (7%) Consider a queue with n elements. We define the *middle* element of the queue to be its $\lceil n/2 \rceil^{\text{th}}$ element, counting from the front. For example, suppose the queue contains the elements A, B, C from front to end, then the middle element is B. If the queue contains the elements D, E, F, G from front to end, then the middle element is E. Note that when we perform the usual **ENQUEUE** or **DEQUEUE** operation, the middle element may change. Suppose we want to implement a special queue that supports, apart from the usual **ENQUEUE** and **DEQUEUE**, the following operation:

EXTRACT_MID: Remove the middle element from the queue.

For example, after **EXTRACT_MID** in the above queue with D, E, F, G, the element E is extracted and the queue will contain D, F, G from front to end. Give an efficient implementation of the above special queue, so that each operation **ENQUEUE**, **DEQUEUE**, and **EXTRACT_MID** can be performed in $O(1)$ time.

8. (6%) Among 150 students, 70 students are wearing hats, 60 are wearing gloves, and 37 are wearing both hats and gloves. Of the 65 students who are wearing sweaters, 32 are wearing hats, 28 are wearing gloves, and 17 are wearing both hats and gloves. Everyone wearing neither a hat nor gloves is wearing scarves.

(a) (2%) How many students are wearing scarves?

(b) (2%) How many students not wearing a sweater are wearing hats but not gloves?

(c) (2%) How many students not wearing a sweater are wearing neither a hat nor gloves?

9. (6%) Consider the equation $x_1 + x_2 + x_3 + x_4 = 13$.

(a) (2%) How many different integer solutions to the equation in which $x_i \geq 1$, $i = 1, 2, 3, 4$? (Note that the two solutions $x_1 = 5, x_2 = 4, x_3 = 3, x_4 = 1$ and $x_1 = 1, x_2 = 4, x_3 = 5, x_4 = 3$ are assumed to be different.)

(b) (4%) How many distinct integer solutions in which $1 \leq x_i \leq 4$, $i = 1, 2, 3, 4$?

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共 5 頁，第 4 頁 *請在【答案卷、卡】作答

10. (7%) Given the set $A = \{1, 2, 3, 4, 6, 12\}$, consider the partial ordering relation R on A .

$$R = \{(x, y) \mid x \text{ divides } y\}.$$

(a) (3%) Draw the Hasse diagram for the partially ordered set (A, R) .

(b) (4%) What is the length of the longest chain in the partially ordered set (A, R) ?

Suppose the length of the longest chains in (A, R) is n , then the elements in A can be partitioned into n disjoint antichains. Please find one such partition.

11. (6%) In a graph G , a set $I \subseteq V(G)$ is an independent set if vertices in I are pairwise nonadjacent, and a set $D \subseteq V(G)$ is a dominating set if every vertex not in D has a neighbor in D . Show that for any graph, the minimum size of a dominating set is not greater than the maximum size of an independent set.

12. (6%) In a type-2 grammar, every production is of the form $A \rightarrow \alpha$, where A is a single non-terminal, and α is a string consisting of terminals and non-terminals. Please find a type-2 grammar for $L = \{a^n b^m c^k : k = |n - m|\}$.

13. (7%) How many nonidentical (though some may be isomorphic) spanning trees are there for the following subgraphs of G (shown in Fig. 1)?

(a) (3%) G_1 : the subgraph of G induced by the vertices $a, b, c,$ and d .

(b) (4%) G_2 : the subgraph of G obtained by removing edge (c, h) from G .

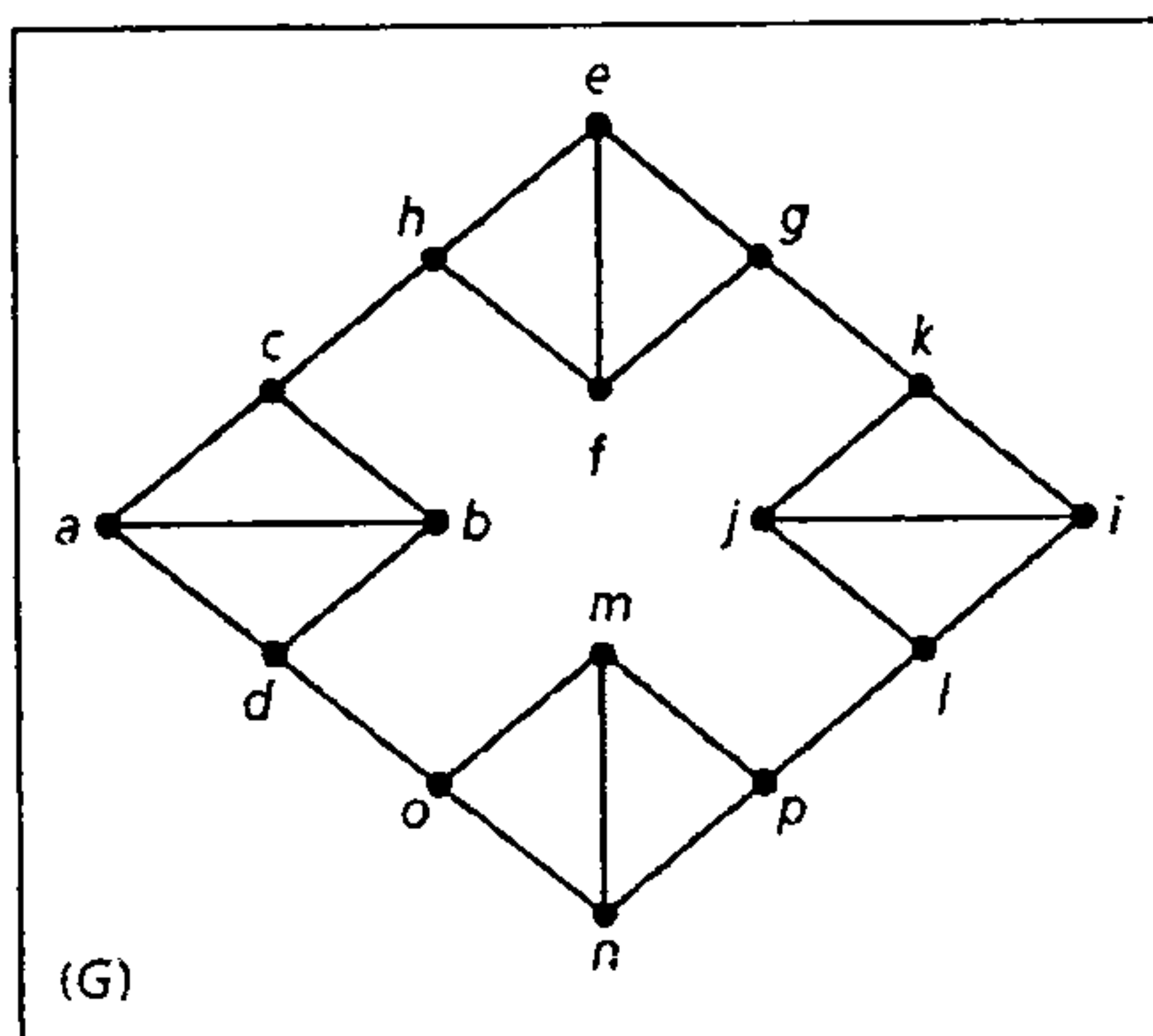


Fig. 1.

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14. (6%) Let a_r denote the number of ways of selecting r objects from n objects with unlimited repetitions.
- (a) (3%) Since each object can be selected as many times as we wish, a_r is equal to the coefficient of z^r in _____.
- (b) (3%) The contribution from each of the above factors will correspond to the number of times one of the objects is selected. Prove that $a_r = C(n+r-1, r)$.
15. (6%) There are two kinds of particles inside a nuclear reactor. In every second, an α particle will split into three β particles, and a β particle will split into an α particle and two β particles. If there is a single α particle in the reactor at $t = 0$, how many particles are there altogether at $t = 100$?