

國立清華大學 命題紙

甲(分子生醫光電)

98 學年度生醫工程與環境科學系(所) 丙(醫學物理與工程)組碩士班入學考試

2303

科目應用數學 科目代碼 2502 共 2 第 1 頁 *請在【答案卷卡】內作答

1. Find the general solution of

$$\mathbf{X}' = \begin{pmatrix} 1 & 8 \\ 1 & -1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} e^{-t} \\ te^t \end{pmatrix} \quad (10\%)$$

2. For the 3x3 matrix

$$\mathbf{A} = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 5 & -1 \\ 0 & 1 & -3 \end{bmatrix}$$

(a) Find the eigenvalues and eigenvectors of \mathbf{A} . (10%)

(b) Find \mathbf{P} and \mathbf{P}^{-1} and \mathbf{D} , so that \mathbf{A} can be formed as \mathbf{PDP}^{-1} , where \mathbf{D} is a diagonal matrix. (5%)

3. For the **series solution** about the point $x=0$ of the following equation,

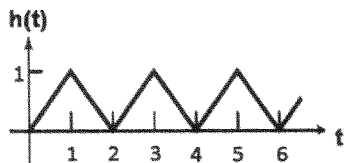
$$2x^2y'' - xy' - 2xy = 0$$

(a) What are the roots of the indicial equation? (5%)

(b) Find the first 5 non-zero terms of two linear independent power series solutions. (5%)

4. (a) Prove that the Laplace transform of a periodic function $f(t)$ with period T can be obtained by integration of $f(t)$ over one period. (5%)

(b) From the result of (a), find the Laplace transform of the following periodic function $h(t)$: (10%)



5. Find the general solution of the following differential equation: (10%)

$$x^3y''' + 4x^2y'' + 11xy' + 9y = 0$$

6. For the differential equation:

$$\left(\frac{2}{x} e^{xy} + ye^{xy} + 3 \sin y + \frac{1}{x} \right) dx + (xe^{xy} + x \cos y) dy = 0$$

(a) Find the integrating factor. (5%)

(b) Find the solution of this equation. (5%)

7. Evaluate the given integral along the indicated closed contours.

$$\oint_C \frac{1}{z^3(z-4)} dz$$

(a) $|z| = 1$. (5%)

(b) $|z-2| = 1$ (5%)

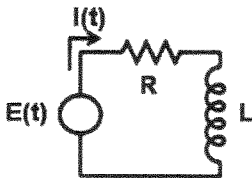
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8. Calculate the current $I(t)$ in the RL circuit of below figure if the current is initially zero and the voltage source $E(t) = \begin{cases} 0 & \text{for } 0 \leq t < 4 \\ 3 & \text{for } t \geq 4. \end{cases}$ (Hint: resistor voltage $V_R(t) = RI(t)$, inductor voltage $V_L(t) = L I'(t)$) (10%)



9. Because a new medical procedure has been shown to be effective in the early detection of an illness, a medical screening of the population is proposed. The probability that the test correctly identifies someone with the illness as positive is 0.99, and the probability that the test correctly identifies someone without the illness as negative is 0.95. The incidence of the illness in the general population is 0.001. You take the test, and the result is positive. What's the probability that you have the illness? (10%)