

國立清華大學命題紙

99 學年度 生醫工程與環境科學系甲組(分子生醫光電組) 碩士班入學考試

科目 應用數學 科目代碼 2303 共 1 頁, 第 1 頁 *請在【答案卷卡】作答

1. Solve the following differential equations. (30%)

(A) $y' = \frac{1-2y-4x}{1+y+2x}$

(B) $2xyy' + (x-1)y^2 = x^2e^x$

(C) $y'' - 3y' + 2y = e^x$

2. Solve the following differential equations by means of Laplace transforms exclusively. (20%)

(A) $y'' - 2y' - 8y = 0$ $y(0) = 3$, $y'(0) = 6$

(B) A linear system

$$y_1' + y_2 = 2[1 - u(t - 2\pi)] \cos t$$

$$y_1 + y_2' = 0$$

$$y_1(0) = 0, \quad y_2(0) = 1$$

3. Find the inverse (A^{-1}) of the following matrix. (Show the details of your work) (10%)

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

4. Find the Eigenvalues and linearly independent Eigenvectors of the following matrix. (Show the details of your work) (10%)

$$\begin{bmatrix} -10 & 10 & -15 \\ 10 & 5 & -30 \\ 5 & -10 & 0 \end{bmatrix}$$

5. Find the angle between two planes, namely, $x - y = 0$ and $x - z = 1$ in the Cartesian coordinate system. (10%)

6. Find the Fourier series of the following periodic function. (10%)

$$f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases} \quad \text{and} \quad f(x + 2\pi) = f(x)$$

7. Find the parametric representation ($\mathbf{r}(t) = [x(t), y(t), z(t)]$) of the three-dimensional curve:

$$y^2 + (z - 3)^2 = 9 \quad \text{and} \quad x = 0. \quad (10\%)$$