國立清華大學 100 學年度碩士班入學考試試題

系所班組別：生醫工程與環境科學系(0525) 甲組(分子生醫光電組)

考試科目（代碼）：應用數學(2503)

Solve the following ordinary differential equations (Problems 1-3).

1. $y\ln y dx + (x - \ln y)dy = 0$ (8 pts)
2. $xy' = y\ln(xy)$ (8 pts)
3. $y'' - 2y' + 5y = e^x \cos 2x$ (8 pts)

Solve the following ordinary differential equations by Laplace transform only (problems 4 and 5).

4. $y'' + y = f(t)$ $y(0) = 0,$ $y'(0) = 1$ (8 pts)

$$f(t) = \begin{cases} 0 & 0<t<\pi \\ 1 & \pi<t<2\pi \\ 0 & t>2\pi \end{cases}$$

5. A differential equation system

$$\frac{dx}{dt} = 4x - 2y + 2u(t-1) \quad \text{(10 pts)}$$

$$\frac{dy}{dt} = 3x - y + u(t-1)$$

$$x(0) = 0, \quad y(0) = \frac{1}{2}$$

6. Find the point on the surface $Z = x^2 + y^2$ at which the gradient is parallel to the vector $4\boldsymbol{i} + \boldsymbol{j} + \frac{1}{2}\boldsymbol{k}$ (8 pts)

7. Find the eigenvalues and normalized eigenvectors of the given matrix

$$\begin{pmatrix} 1 & 6 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix} \quad \text{(8 pts)}$$

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8. Find a 2x2 matrix that has eigenvalues 2 and 3 and the corresponding eigenvectors $\begin{pmatrix}1\\2\end{pmatrix}$ and $\begin{pmatrix}1\\1\end{pmatrix}$ (8 pts)

9. $f(x)=\begin{cases}-\pi, & -2\pi < x < -\pi \\ x, & -\pi < x < \pi \\ \pi, & \pi < x < 2\pi\end{cases}$, expand f(x) in an appropriate Fourier sine **or** cosine series. (8 pts)

10. Proof that the Fourier transform of a real odd function is an imaginary function. (8 pts)

11. Solve the wave equation $4\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ of a string of length of 3 subject to the given condition: u(0,t)=0, u(3,t)=0, u(x,0)=0, $\frac{\partial u}{\partial t}|_{t=0} = 2\sin\frac{\pi}{3}x + \sin\pi x$ (10 pts)

12. $\mathbf{F} = y^2\mathbf{i} + xz^3\mathbf{j} + (z-1)^2\mathbf{k}$; Use the divergence theorem to find the outward flux of the region bounded by cylinder $x^2 + y^2 = 16$ and the planes z=1, z=5. (8 pts)