台灣聯合大學系統97學年度碩士班考試命題紙

科目：應用數學(3001)　校系所組：中大物理學系、天文研究所

交大電子物理學系丙組、物理研究所

清大物理學系、先進光源科技碩士學位學程甲組、天文研究所

陽明生物醫學影像暨放射科學系生物醫學影像組

陽明生醫光電工程研究所理工組A

Applied Mathematics

Show your calculation steps clearly.

Problem 1 (20 points)

(1a) Write down the definition of a group. ($\sim$ 4 points)

(1b) Let U be a group element of the unitary group $U(3,C)$ (also written as $U(3)$) and define G by

$$U \equiv \exp[G] \equiv \sum_{n=0}^{\infty} \frac{G^n}{n!},$$

write down a set of complete basis { T_i } that spans the space of Lie algebra $u(3,C)$. [Hint: $G = \sum_{i=1}^{N} G_i T_i, \forall G \in u(3,C)$ and $G_i \in \mathrm{R}$. You must answer $N =$? and list the properties of G.] ($\sim$ 8 points)

(1c) For a set of nonabelian operators A, B, and C, $[A,\{B,C\}] = [\{A,B\},C] - [B,\{A,C\}]$ is known as the Jacobi identity. Show that there is another expression for the Jacobi identity : $[A,\{B,C\}] = \{[A,B],C\} + Y$ by expressing Y as similar linear combination of commutators ([,]) and/or anticommutators ({ , }) of A, B, and C. [Hint: Y must be derived or be provided with a geometric (or statistical) interpretation, not just writing it down out of your memory.] ($\sim$ 8 points)

Problem 2 (15 points)

Show that

$$\pi = \sum_{n=0}^{\infty} (-)^n \frac{4}{2n+1}$$

by integrating $\int_0^? f(x)dx$ with a corresponding function $f(x)$. [Hint: $f(x)$ must be derived from converting the series into an integral. 7 points if you derive a compact expression for $f(x)$.]

Problem 3 (10 points)

Let $\Omega = \frac{N!}{n_1! n_2! \dots n_K!} g_1^{n_1} \dots g_K^{n_K}$ where g_i's are constants. Here $N, n_i \gg 1$. Assume there exist two constraints $\sum n_i = N$ and $\sum \epsilon_i n_i = E$ where N and E are two constants. Show that $n_i = g_i C e^{-\beta \epsilon_i}$ when Ω is at a maximum (C and β are constants). (*Hint*: Use the equivalent condition that $\ln \Omega$ is at a maximum. $\ln N! \approx N \ln N - N$ for large N.)

注意：背面有試題

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參考用

Problem 4 (10 points)

(4a) For a contour map $h = 0.001x^2 + 0.002y^2$, find the change of height δh for the displacement $\delta \mathbf{r} = (2,1)$ at $\mathbf{r} = (x=100, y=300)$. (4 points)

(4b) Along what direction is the largest slope at this position? (3 points)

(4c) Show that ∇h is always perpendicular to the constant h contour. (3 points)

Problem 5 (10 points)

(5a) For $N = ak^2$ and $\omega = \sin(bk)$ where a and b are constants, find $\frac{dN}{d\omega}$.

(5b) If $f(x)=3x+5$ and $g(x^2\text{-}1)=f(x-7)$, find the value of $g(6)$. (5 points each)

Problem 6 (35 points)

(6a) Discuss the analytic properties of the following functions in the complex z-plane and, if they have singularities, state the nature of the singularity (branch points, poles, etc) (12 points)

(i) e^z ;

(ii) $\frac{1}{\sqrt{z-i}}$;

(iii) z^* .

(6b) Use Cauchy's theorem to evaluate the following contour integral where C is the circle (i) $|z| = 2$;(ii) $|z| = \frac{1}{2}$. (12 points)

$$\oint_C \frac{e^z}{(z-1)^2} dz.$$

(6c) Use the method of Fourier transform to solve

$$\frac{d^2y}{dx^2} - y = -\theta(1-|x|)$$

for $-\infty < x < \infty$, with $y(x) \to 0$ and $\frac{dy}{dx} \to 0$ as $|x| \to \infty$. Note that

$$\theta(x) = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x < 0 \end{cases}$$

is the Heaviside function. (11 points)