

國立清華大學命題紙

99 學年度核子工程與科學研究所甲組、聯合招生(工科丙組、先進光源工科組)碩士班入學考試  
科目 近代物理 科目代碼 2803、9802 共 / 頁，第 / 頁 \*請在【答案卷卡】作答

Useful constants:  $c = 3.00 \times 10^8$  m/s,  $k_B = 1.38 \times 10^{-23}$  J/K,  $h = 6.63 \times 10^{-34}$  J·s,  $e = 1.60 \times 10^{-19}$  C,  
 $m_e = 9.11 \times 10^{-31}$  Kg,  $\mu_B = 9.274 \times 10^{-24}$  J/T,  $R = 1.097 \times 10^7$  m<sup>-1</sup>.

1. (15%) An unstable particle having a mass of  $3.34 \times 10^{-27}$  kg is initially at rest. The particle decays into two fragments that fly away with speeds of  $0.987c$  and  $0.868c$ , respectively. Find the rest masses of these two fragments.
2. (15%) Photons of wavelength 450 nm are incident on a metal. The most energetic electrons ejected from the metal are bent into a circular arc of radius 20 cm by a magnetic field whose strength is equal to  $2.0 \times 10^{-5}$  T. What is the work function of the metal?
3. (20%) Derive the Energy levels of hydrogen according to Bohr's quantum model of the atoms.
4. (20%) For a simple harmonic oscillator of spring constant  $K$  and mass  $m$ , one solution of the Schrödinger equation is of the form  $\psi(x) = Ae^{-ax^2}$ .  
(A) Determine the energy of this state, and the normalization constant  $A$ .  
(B) Find the uncertainty in position  $\Delta x$ , and the uncertainty in momentum  $\Delta p$ .
5. (15%) An electron in a hydrogen atom is in a 3d state.  
(A) What is the minimum angle the angular momentum vector may make with the z-axis?  
(B) What is the most probable radius at which to find it? Given that  $R_{32}(r) = Ar^2e^{-r/3a_0}$ .  
(C) Make a plot to show the splitting of the 3d state due to the spin-orbit interaction.
6. (15%) Two noninteracting identical particles in a one-dimensional box  $(0, L)$  occupy the  $n = 1$  and  $n' = 2$  single-particle states.  
(A) Write down the normalized single-particle wave function  $\psi_n(x)$ .  
(B) Write down the symmetric and antisymmetric wave functions:  $\psi_s(x_1, x_2)$  and  $\psi_A(x_1, x_2)$  for the two-particle system.  
(C) Consider the two electrons in the ground state helium atom. If we neglect the interaction between the electrons, then the single-particle state can be described by the set of quantum numbers:  $\{n, l, m_l, m_s\}$ . Write down the system wave function of the ground state helium atom.