

科目：應用數學(3001)

校系所組：中央大學照明與顯示科技研究所

交通大學電子物理學系 (丙組)

交通大學物理研究所

清華大學物理學系

清華大學先進光源科技學位學程

陽明大學生醫光電研究所 (理工組)

中央大學天文研究所

清華大學天文研究所

No point will be graded unless logic reason is written down explicitly for every single step throughout the derivation. Do not just copy identities without reasoning.

- 1 Let  $O$  be a physical observable having real expectation value for all physical states  $|\psi\rangle$ , derive the explicit relation between  $O$  and its complex conjugate  $O^*$  from the reality condition. (4%)

Problem [2]-[4] is a problem set for the same hermitian matrix  $A$ . Let  $\lambda$  and  $|\psi\rangle = \begin{pmatrix} l \\ m \\ n \end{pmatrix}$  be the eigenvalue

and eigenvector associated with the  $3 \times 3$  hermitian matrix  $A = \begin{pmatrix} a & d & ? \\ ? & b & f \\ e & ? & c \end{pmatrix}$ .

- 2 Write down (a) the complete matrix  $A$  with all required constraints. (b) Derive further the inverse matrix  $A^{-1}(=?)$  of  $A$ . (8%)

- 3 Let  $\alpha = \det A$ ,  $\beta = \text{Tr } A$  and  $\gamma = \text{Tr } A^{-1}$ , (a) show that the eigenvalue equation can be written as an algebraic equation of degree 3:  $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$ . (b) Write  $f = a_1 + a_2 + a_3$  as a function of  $\alpha$ ,  $\beta$  and  $\gamma$ . (c) Show in details whether the parameter  $f$  is invariant under the diagonalization process. (12%)

- 4 If one of the eigenvalue is know to be  $\lambda$  and  $n = \lambda^2 - (a+b)\lambda + ab - |d|^2 \neq 0$ , derive  $l$  and  $m$  as functions of  $\lambda, a, b, c, d, e$ , and  $f$ . (10%)

- 5 Solve the following equation. (8%)

$$\frac{dy}{dx} = \frac{5x^4 + 6x^3 + 15x^2 + 14x + 12}{x^5 + x^4 + 5x^3 + 5x^2 + 6x + 6}$$

- 6 Use Laplace transform to solve the following equation with  $y'(0) = y(0) = 0$ . (10%)

$$y'' + 9y - \sin(3t) = 0,$$

- 7 Solve the following differential equation system with the initial values  $(x, y) = (1/2, 1)$ . (15%)

$$\begin{cases} \frac{dx}{dt} = 2x + y - 1 - e^t \\ \frac{dy}{dt} = y + e^t \end{cases}$$

- 8 The coordinates of two points  $A$  and  $B$  on the plane  $R^2$  are  $(1, \frac{1}{2})$  and  $(1 + \sqrt{2}, \frac{1+\sqrt{6}}{2})$ , respectively. Calculate the coordinate of the third point  $C$  on the plane so that  $\triangle ABC$  forms an equilateral triangle. (11%)

- 9 Do the integration  $\int_0^{2\pi} \frac{d\theta}{\sqrt{2-\cos\theta}} = ?$ . (11%)

- 10 Let  $f(z) = u(x, y) + iv(x, y)$  be an analytic function on the whole complex plane, and  $u(x, y) = x^2 - y^2 - x$ . Calculate  $f(z) = ?$  (11%)