Existence of solitary waves in optical fibers owing to the mutual support between bright and dark pulses

Likarn Wang and C. C. Yang

Communications and Space Sciences Laboratory, Department of Electrical Engineering, The Pennsylvania State University, University Park, Pennsylvania 16802

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We introduce a new type of solitary wave in a lossless single-mode fiber in which a bright solitonlike pulse in the anomalous-dispersion range propagates together with a stationary dark pulse in the normal-dispersion range. Such a solitary wave can be generated by launching a bright pulse in the anomalous-dispersion range together with a long pulse of the same speed in the normal-dispersion range into a fiber. The results of numerical simulations show that the dark pulse is not a dark soliton. The generation and maintenance of such a solitary wave are attributed to the mutual chirping through cross-phase modulation between those two waves in the anomalous- and normal-dispersion ranges.

The existence of the optical solitons in a single-mode fiber was first predicted by Hasegawa and Tappert in 1973.\(^1\) In a single-mode fiber the interaction between self-phase modulation and anomalous dispersion (AD) leads to the existence of bright solitons; that between self-phase modulation and normal dispersion (ND) leads to the formation of dark solitons. In 1980 bright solitons in optical fibers were experimentally observed for the first time by Mollenauer et al.\(^2\) This observation has brought about a series of theoretical studies and experiments aimed at the implementation of soliton-based fiber communications. Stable propagation of fundamental bright solitons over 6000 km has been observed by Mollenauer and Smith.\(^3\) However, optical dark solitons in optical fibers were not experimentally observed until recently because of the difficulty in generating the initial narrow dark pulses.\(^4\)\(^5\)

Use of a pulse to create another pulse or to support the stable propagation of another pulse through cross-phase modulation (XPM) in a single-mode fiber has been reported.\(^6\)\(^7\)\(^8\) A bright pulse in the ND range and a dark pulse in the AD range can support each other through XPM to form a solitary wave.\(^6\) The mutual support of a bright and a dark soliton in the AD range can also lead to stationary propagation.\(^7\) In addition, a bright pulse in the ND range can generate another bright pulse on a cw background in the AD range.\(^8\) In this Letter we introduce the generation of a new type of solitary wave. The results of our numerical simulations show that a dark pulse is generated on a long super-Gaussian pulse in the ND range when this super-Gaussian pulse co-propagates with a bright pulse of the same speed in the AD range in a single-mode fiber. This dark pulse can propagate together with the bright pulse, which is somewhat modified, to form a solitary wave. Apparently the generated dark pulse in the ND range and the modified bright pulse in the AD range support each other through XPM. Mutual support of a dark soliton in the ND range and a bright soliton in the AD range in a nonlinear medium has been discussed and referred to as the noninverted case.\(^9\)\(^10\) However, the noninverted case cannot be applied to pulse propagation in a single-mode optical fiber.\(^10\) Our individual bright and dark pulses are not solitons, as is discussed in the following. Therefore the solitary wave that we present here must be different from those in the noninverted case.

In a lossless fiber, the pulse envelope in the AD range \(u\) and the pulse envelope in the ND range \(v\) are governed by two coupled nonlinear Schrödinger equations:

\[
\frac{1}{2} \frac{\partial u}{\partial \tau} + \frac{\partial^2 u}{\partial x^2} + x_u (|u|^2 + 2|v|^2) = 0 \quad (1)
\]

and

\[
\frac{1}{2} \frac{\partial v}{\partial \tau} + j \frac{\partial v}{\partial x} - \beta \frac{\partial^2 v}{\partial x^2} + x_v (|v|^2 + 2|u|^2) = 0. \quad (2)
\]

Here \(\tau\) and \(x\) are the normalized distance and normalized retarded time, respectively; \(\beta\) denotes the difference of the group velocities between the two waves; and \(\beta = |k_{ov}/k_{ou}|\), with \(k_{ou}\) and \(k_{ov}\) representing the second-order derivatives of the wave number \(k\) at the carrier frequencies of \(v\) and \(u\), respectively. Note that \(k_{ou} < 0\) and \(k_{ov} > 0\) because the pulses \(u\) and \(v\) are in the AD and ND ranges, respectively. Also in Eqs. (1) and (2), \(x_u = 2k_{ou} / (k_{ou} + k_{ov})\) and \(x_v = 2k_{ov} / (k_{ou} + k_{ov})\), with \(k_{ou}\) and \(k_{ov}\) representing the wave numbers at the carrier frequencies of the pulses \(u\) and \(v\), respectively.

Equations (1) and (2) are solved numerically by using the beam-propagation method with the following initial conditions:

\[
u(t = 0, \tau) = (u_0 / \sqrt{\pi}) \cdot \text{sech}(\tau / \tau_0)
\]

and

\[
u(t = 0, \tau) = \exp[-(\tau / 40)^4].
\]

Here \(\tau_0\) is always chosen to be much smaller than 40. Note that when \(\omega_0 = 1 / \tau_0\), Eq. (3) represents a bright one-soliton pulse. In our numerical simulations, we arbitrarily assume that there exists a single-mode fi-
ber such that the wavelengths 1.2 and 1.35 μm belong to the ND and AD ranges, respectively, and \( \beta = 0.8744 \), \( x_u = 0.9406 \), and \( x_v = 1.0594 \). It is also assumed that the group velocities at these two wavelengths are the same, and hence \( \delta = 0 \). Note that, as long as \( \delta = 0 \), changes of the above input parameters will not affect the physical pictures in the following discussions.

First, we set \( u_0 = 1/\tau_0 = 2 \) in Eq. (3). The evolution of the bright soliton \( u \) up to a distance 2π is shown in Fig. 1(a). The pulse shape of \( v \) at the distance 2π is shown in Fig. 1(b). In this situation, we can observe that the bright soliton is destroyed and that no stationary dark pulse is generated on the long super-Gaussian pulse. However, if we increase \( u_0 \) to \( u_0 = 1/\tau_0 = 3 \) in Eq. (3), the evolution is completely different. From Fig. 2(a) it can be seen that the propagation of the bright pulse in the AD range is stable up to a distance of 2π, although it experiences some modifications. We also observe that a dark pulse is generated on the long super-Gaussian pulse. This dark pulse in the ND range propagates almost without distortion after the distance π. The pulse shape of this dark pulse and its background at the distance 2π is shown in Fig. 2(b). Tests indicate that this dark pulse and the above-mentioned bright pulse will propagate together without any distortion and form a solitary wave. To see if the bright and dark pulses in this solitary wave are a bright soliton and a dark soliton, respectively, we make comparisons in pulse shapes at the distance of 2π in Fig. 2(c). Here we plot a dashed curve for a bright one-soliton pulse to fit the central part of the

![Figure 1](image1.png)

**Fig. 1.** Numerical results when a bright one-soliton pulse \( u(\tau = 0, \tau) = 2.062 \text{ sech}(2\tau) \) is used at the input end. (a) Evolution of the bright pulse up to the distance of 2π; the distance between any two successive curves is π/2. (b) The shape of the dark pulse at the distance of 2π.

![Figure 2](image2.png)

**Fig. 2.** Numerical results when a bright one-soliton pulse \( u(\tau = 0, \tau) = 3.093 \text{ sech}(3\tau) \) is used at the input end. (a) Evolution of the bright pulse up to the distance of 2π; the distance between any two successive curves is π/2. (b) The shape of the dark pulse at the distance of 2π. (c) Comparison of the central parts of the bright and dark pulses at 2π in the numerical simulations (the solid curves) with a bright and a dark soliton (the dashed curves). (d) The phases of the bright (curve I) and dark (curve II) pulses at 2π in the numerical simulations; the dashed curve represents the phase of the dark soliton plotted in (c).
bright pulse in our simulation data, which is plotted as the corresponding solid curve. Except for the tails, the dashed and solid curves are highly coincident. However, the designated dark soliton, plotted as another dashed curve, does not fit the dark pulse in our results well. The dark pulse in our results is broader than the dark soliton. Further comparisons in phases are demonstrated in Fig. 2(d). Here solid curves I and II represent the phases of the bright and dark pulses in our simulations, respectively, and the dashed curve represents the phase of the dark soliton plotted in Fig. 2(c). We can see that the phases of both bright and dark pulses are flat in the regions of significant power. This indicates that the bright pulse in our simulation results is close to a bright soliton; however, the dark pulse is not a dark soliton. The stationary propagation of the dark pulse is supported by the chirping from the bright pulse through XPM; otherwise, an isolated dark pulse with a constant phase will evolve into a pair of dark solitons. On the other hand, the dark pulse also provides the bright pulse with chirping through XPM such that the bright pulse, which is not a bright soliton, can propagate without distortion.

From these two sets of data, we can conclude that a solitary wave, including a bright soliton-like pulse and a dark pulse, can be generated only when the input bright pulse is sharp enough. Essentially, in the early stage of evolution the bright pulse produces a chirping through XPM on the long super-Gaussian pulse. This chirping will evolve into amplitude modulation on the long pulse owing to frequency dispersion. This amplitude modulation will provide the bright pulse with a feedback chirping again through XPM. This feedback chirping has a tendency to destroy the bright pulse. If the initial bright pulse is sharp enough, it will reshape itself to form another soliton-like bright pulse, as shown in Fig. 2(a); otherwise, the bright pulse is destroyed and forms a multihumped broad pulse, as shown in Fig. 1(a). As long as the sharp bright pulse can be preserved, it will help the above-mentioned amplitude modulation on the long super-Gaussian pulse to evolve into a sharp dark pulse at the center and in the oscillatory tails. Eventually, the bright pulse in the AD range and the sharp dark pulse in the ND range will form a solitary wave. The reason that the bright pulse is so close to a bright one-soliton pulse but the dark pulse is so different from any dark solitons deserves more investigation.

A phenomenon in the copropagation of a bright pulse in the AD range and a long pulse in the ND range was discussed in Ref. 6. In the simulation results of that study, an input Gaussian pulse in the AD range was compressed to form a bright two-soliton pulse. In addition, a broad dip was formed at the center of the long pulse. In other words, no solitary waves as in our results were observed. We have tried the following inputs:

\[ u(\xi = 0, \tau) = \exp\left[-(\tau/\tau_1)^2\right] \]  

and

\[ v(\xi = 0, \tau) = \exp\left[-(\tau/15)^3\right], \]

which are similar to those in Ref. 6. When \( \tau_1 = 7.07 \), our simulation results up to the distance of 4\( \pi \) are similar to those in Ref. 6. However, when \( \tau_1 \) is reduced to 5, i.e., the pulse becomes sharper, we observe that a bright pulse close to a one-soliton pulse is formed after a distance of 3\( \pi \), and a sharp dark pulse is generated on the long pulse. Again, a solitary wave is formed. These results not only imply that a bright pulse other than a soliton in the AD range can be used for generating a solitary wave but also confirm that such a bright pulse must be sharp enough to produce a solitary wave.

In summary, we have numerically demonstrated the existence of a solitary wave in which a bright pulse in the AD range and a dark pulse in the ND range can propagate together. Such a solitary wave can be generated simply by launching a sharp enough bright pulse in the AD range together with a long pulse of the same speed in the ND range. In such a solitary wave, the bright pulse closely resembles a bright soliton; however, the dark pulse is different from any dark solitons.

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References