1. INTRODUCTION

Single-mode oscillation is a desirable attribute for a laser source. However, in a laser resonator, several transverse and longitudinal modes can oscillate simultaneously in the laser gain bandwidth. Usually it is not difficult to remove high-order transverse modes by limiting the gain aperture or ensuring good laser alignment. To achieve single-longitudinal-mode oscillation, one often has to insert a dispersive element, such as an etalon or a grating, into the laser cavity. The etalon introduces a mode-dependent loss, so only one mode is present in the laser gain bandwidth. An optical grating selectively reflects optical wavelengths that satisfy the Bragg condition. When it is built into a laser gain medium, a Bragg grating provides distributed optical feedback similarly to a resonator mirror and remarkably gives a mode-dependent laser threshold. If the laser medium is homogeneously gain broadened, only the lowest-threshold mode oscillates at the steady state. Inasmuch as a Bragg grating can be monolithically integrated into a diode laser during microfabrication, single-frequency distributed-feedback (DFB) diode lasers have been used in numerous applications. In a nonlinear frequency-conversion process, the wavelengths that are generated are no longer limited to the energy levels of the source material. For example, an optical parametric oscillator (OPO) produces two low-frequency photons through a pump photon with the sum of the two output photon frequencies equal to the pump photon frequency. Because an optical parametric process permits wavelength tuning, it is advantageous to extend the study of a DFB laser to a DFB OPO. Tentatively, the Bragg grating in a DFB OPO is used for resonating one of the output fields near Bragg resonance.

The use of a Bragg grating in conjunction with a nonlinear optical material has been discussed in the literature. Previous publications reported wave propagation in a Bragg grating with a laser-intensity-dependent refractive index, which led to several interesting phenomena, including optical bistability, self-pulsing, chaos, pulse compression, and gap solitons. Periodic dielectric modulation was also employed previously to achieve quasi-phase matching in harmonic generation in nonlinear optical materials. However, the purpose of the dielectric modulation studied in this paper is not directly related to the aforementioned applications but is related to grating-scattering enhanced local optical fields to improve nonlinear frequency conversion efficiency or achieve optical oscillation in a nonlinear optical material. Grating-enhanced second-harmonic generation has been studied extensively with an optical grating built in a nonlinear material strongly scattering both the fundamental and second-harmonic field. Recently second-harmonic generation was cited as an example of grating-enhanced or -suppressed stimulated emission in a one-dimensional photonic crystal. To the best of our knowledge there has been no report of the use of coherent grating scattering for nonlinear frequency downconversions such as optical parametric amplification and oscillation. As will be seen below, a DFB optical parametric amplifier (OPA), like grating second-harmonic generation; also derives improved efficiency from the strong localized optical fields in a grating structure. In addition, a DFB OPO has the advantage of both mode selectivity and wavelength selectivity. Although some preliminary evidence of optical parametric oscillation was recently observed in a periodically poled lithium niobate crystal with a built-in photorefractive DFB grating, a full theoretical model and physical understanding have yet to be developed for such a coherent light source.
A DFB OPO is fundamentally different from a DFB diode laser in several respects: First, most DFB diode lasers are electrically pumped and fixed in a small frequency bandwidth from the atomic energy levels, whereas a DFB OPO is optically pumped and wavelength selectable in a phase-matching bandwidth. A DFB diode laser has optical gain in both longitudinal directions, whereas a DFB OPO has gain in only the phase-matched direction. A DFB diode laser has only one optical wave in the gain medium, whereas a DFB OPA has three, including pump, signal, and idler waves. Compared to a DFB laser amplifier, a DFB OPA has one more freedom in selecting the seeding optical wave. Usually it is difficult for a DFB laser to function as an amplifier because of the high injection loss and reflection feedback to the seeding source near the Bragg resonance. One can achieve a DFB OPA, however, by seeding the nonresonance wave while producing the resonance wave near the Bragg wavelength. Our goal in this paper is to develop a theoretical model for this new class of coherent light source, calculate the amplification gain of a DFB OPA from theory, and prove longitudinal-mode selectivity in a DFB OPO under various boundary conditions. In an optical parametric process the high-frequency output photon is usually called the signal output and the lower-frequency photon is called the idler output. In a DFB OPO, either of the two output fields could oscillate in the DFB structure. To facilitate the discussion in this paper, we call the photon resonating in the DFB structure the signal and the other photon the idler.

This paper is organized as follows: In Section 2 we develop the coupled-wave theory for pump, signal, and idler waves in a dielectric modulated nonlinear optical material. In Section 3 we derive the analytic field solutions of the signal and idler waves at Bragg resonance for a DFB OPA and compare the results with known properties of an ordinary OPA and a DFB laser. The comparison is followed by numerical calculations of the signal and idler amplification gain detuned from Bragg resonance. In Section 4 we derive the resonance conditions of a DFB OPO subject to various boundary reflections and discuss the mode-dependent threshold gain in such an oscillator. In Section 5 we present the formulation for studying the resonant modes in a cascaded DFB OPO with arbitrary design parameters in each DFB section and phase shifts between adjacent DFB sections. In Section 6, variants of the DFB OPA and OPO are described and possible implementations of the Bragg gratings in nonlinear optical materials are proposed.

A few practical, although not necessarily the most general, assumptions are made in this paper. Because nonlinear frequency conversion is a coherent process, in this paper we assume monochromatic mixing waves. Like a DFB diode laser, a DFB OPA or OPO could be most easily implemented in a nondiffractive waveguide structure. We therefore assume that all the mixing waves are plane waves with slowly varying field envelopes along the Bragg grating-vector direction. Usually it takes time to build up resonance in a resonator. We allow enough time to reach steady-state resonance in the DFB structures and confine the scope of this study to continuous-wave cases.

2. COUPLED-WAVE THEORY

Figure 1 depicts the general device configuration considered in this paper. Forward Bragg wave vector \( \mathbf{\beta}_0 \) associated with the signal wave is collinearly phase matched to the pump and idler wave vectors, \( \mathbf{k}_s \) and \( \mathbf{k}_i \), respectively, in the \( +z \) direction. The dielectric modulation has a spatial period \( \Lambda_g \) in a lossless nonlinear optical material of length \( L \). The pump and idler waves copropagate toward the \( +z \) direction. According to the configuration, the linear dielectric constant in the material can be expressed by a Fourier series given by

\[
\epsilon(z) = \epsilon + \sum_{l \neq 0} \Delta \epsilon_l \exp(-j l k_g z),
\]

(1a)

where \( j = \sqrt{-1} \) is an imaginary unit, \( l \) is an integer other than zero, \( \epsilon \) is the average dielectric constant, \( \Delta \epsilon_l \) is the Fourier amplitude of the dielectric modulation, and \( k_g = 2 \pi/\Lambda_g \) is the DFB grating vector in the \( z \) direction.

With weak dielectric modulation \( \Delta \epsilon_l \ll \epsilon \), the refractive index \( n(z) \) calculated from Eq. (1a) is given by

\[
n(z) = n + \sum_{l \neq 0} \delta n_l \exp(-j l k_g z),
\]

(1b)

where \( n = \sqrt{\epsilon}, \Delta n_l = \Delta \epsilon_l/2\epsilon, \) and \( \Delta n_l \ll n \).

Under the plane-wave assumption, one can express the electric fields of the pump, signal, and idler waves in an optical parametric process by

\[
\vec{E}_{p,s,i}(z, t) = \text{Re}[E_{p,s,i}(z) \exp(j \omega_{p,s,i} t)],
\]

(2a)

where the subscripts \( p, s, \) and \( i \) denote pump, signal, and idler, respectively, \( \omega \) is the angular frequency of the electromagnetic field, and \( E(z) \) is the complex amplitude of the field. It is unlikely that a DFB OPO would operate near the degeneracy for which \( \omega_s = \omega_i = \omega_p/2 \). If the signal–idler wavelength satisfies the first Bragg resonance at degeneracy or \( \lambda_s = \lambda_i = 2 \Lambda_g \), the pump wavelength, \( \lambda_p \sim \Lambda_g \), will be too close to the second Bragg resonance, and pump reflection can occur. For most OPAs and OPOs operating far away from degeneracy, the

![Fig. 1. Configuration of the DFB OPA and OPO discussed in this paper. Forward Bragg wave vector \( \mathbf{\beta}_0 \) associated with the signal wave is collinearly phase matched to pump and idler wave vectors \( \mathbf{k}_s \) and \( \mathbf{k}_i \), respectively, in the \( +z \) direction. The dielectric modulation has period \( \Lambda_g \) in a lossless nonlinear optical material of length \( L \). Normalized coordinate \( \tilde{z} = z/L \) is introduced for subsequent analysis.](image-url)
wavelengths of the mixing waves are fairly different. Here, we define that the signal is the wave in resonance with the DFB structure and that the pump and idler waves are unaffected by the weak dielectric modulation. Therefore the complex amplitudes of the pump and idler waves have only forward-propagation components, given by

\[ E_{p,i}(z) = A_{p,i}(z) \exp(-j k_{p,i} z), \] (2b)

where \( A(z) \) is the slowly varying field envelope within a wavelength, \( k = \omega n / c_0 = 2\pi / \lambda \) is the wave number, and \( \lambda \) is the wavelength in an unperturbed nonlinear medium. It should be noted that the definition of position-independent \( k \) in an unperturbed medium has lumped the effect of the index perturbation into the field envelope, \( A(z) \). As a result of Bragg reflection, the signal wave has both forward- and backward-propagating components, \( E_s(z) \) and \( E_b(z) \), respectively, given by

\[ E_s(z) = E_p(z) + E_b(z) = R(z) \exp(-j \beta_0 z) \]
 \[ + S(z) \exp(j \beta_0 z), \] (2c)

where \( \beta_0 = 2 m_0 / \lambda_0, \) \( \lambda_0 \) is the Bragg wavelength to be determined from the phase-matching condition in Eq. (3) below, and \( R(z) \) and \( S(z) \) are the slowly varying envelope fields of the forward and backward components, respectively. The effective propagation constant for the signal wave, defined in the Helmholtz equation and given by \( \pm \beta = \pm \omega n / c_0 \), where \( c_0 \) is the speed of light in vacuum, is in general slightly different from \( \beta_0 \) for a resonant mode. In other words, \( \beta_0 \) is an unknown parameter in the guessing solution [Eq. (2c)] and will be solved through the phase-matching condition in a DFB structure, whereas \( \beta \) is an intrinsic parameter defined through the wave frequency and the refractive index of the material. In the following calculation, one can see that the dielectric period defines \( \beta_0 \) through the Bragg condition and that the mode resonance determines the frequency in \( \beta_0 \).

For a nonlinear optical material with weak index perturbation, the total mixing field, \( \tilde{E} = \tilde{E}_p + \tilde{E}_s + \tilde{E}_i \), satisfies the wave equation with a driving term equal to nonlinear polarization \( \tilde{P}_{NL}(z, t) \):

\[ \nabla^2 \tilde{E} - \mu_0 c_0^2 \frac{\partial^2 \tilde{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \tilde{P}_{NL}}{\partial t^2}, \] (3)

where \( \varepsilon_0 \) is the vacuum permittivity, \( \mu_0 \) is the same as the vacuum permeability for a nonmagnetic material, \( \tilde{P}_{NL} = 2d \varepsilon_0 \tilde{E}^2 \) is the second-order nonlinear polarization density, and \( d \) is the effective nonlinear coefficient. The choice of \( d \) depends on wave polarizations and propagation directions. We assume that the nonlinear coefficient is unaffected by the index perturbation. Note that for simplicity we have used scalar wave equation (3) for analysis. Not all the wave fields are necessarily polarized in the same direction. Here, we simply label the refractive indices \( n_{p,s,i} \) and the effective nonlinear coefficient \( d \) without deriving their specific expressions for different wave polarizations. In an anisotropic medium one can obtain the specific expressions for the refractive indices and the effective nonlinear coefficient by considering the directions of wave polarization and propagation in conjunction with the index ellipsoid and the nonlinear-coefficient tensor of the material.

Substituting Eqs. (1) and (2) into Eq. (3) and equating terms that satisfy \( \omega_s = \omega_i + \omega_l \), one obtains the following coupled-wave equations under the Bragg condition that \( 2 \beta_0 - lk = 0 \):

\[ \frac{dR(z)}{dz} = -j \delta R(z) - j \kappa S(z), \]

\[ -j \kappa n_{p} A_p(z) A_i^*(z) \exp(-j \Delta k z), \] (4a)

\[ \frac{dS(z)}{dz} = j \kappa R(z) + j \delta S(z), \] (4b)

\[ \frac{dA_p(z)}{dz} = j \kappa n_{p} A_p(z) R(z) \exp(j \Delta k z), \] (4c)

\[ \frac{dA_i(z)}{dz} = -j \kappa A_i(z) R(z) \exp(j \Delta k z), \] (4d)

where \( \Delta k = k_p - \beta_0 - k_i \) is the wave-vector mismatch among the mixing waves, \( \delta = (\beta_0 ^2 - \beta_i ^2) / 2 \beta_0 \approx \beta - \beta_0 \) is the detuning parameter, \( \kappa = 2 \pi \Delta n / \lambda_0 \) is the DFB coupling coefficient for the forward and backward signal waves, and \( \kappa_{p,s,i} = \omega_{p,s,i} d / (n_{p,s,i} c_0) \) are the parametric coupling coefficients for the pump, signal, and idler waves. The Bragg condition \( 2 \beta_0 - lk = 0 \) defines \( \beta_0 \) or the Bragg resonant wavelength \( \lambda_0 = 2 \pi \Lambda / k \). Without the parametric coupling coefficients, \( \kappa_{p,s,i} = 0 \), Eqs. (4) are reduced to the coupled-wave equations for a zero-gain DFB laser.\(^{10}\) Without the DFB coupling coefficient, \( \kappa = 0 \), Eqs. (4) are reduced to the coupled-wave equations for an ordinary OPO.\(^{11}\)

From Eqs. (4) and \( dI/dz \times (E \times dE^*/dz + E^* \times dE/dz)n \), it is straightforward to show that

\[ \frac{1}{\omega_s} \left( \frac{dI_p}{dz} - \frac{dI_i}{dz} \right) = \frac{1}{\omega_p} \frac{dI_i}{dz} - \frac{1}{\omega_p} \frac{dI_p}{dz}, \] (5)

where \( I_p \) is the pump intensity, \( I_i \) is the idler intensity, and \( I_f \) and \( I_b \) are the forward and backward intensities of the signal wave, respectively. Equation (5) is consistent with the Manley–Rowe relation or photon-number conservation in nonlinear frequency conversion.

For the signal and idler waves to be generated efficiently, the phase mismatch among the forward pump, signal, and idler waves,

\[ \Delta k z = (k_p - \beta_0 - k_i) z, \] (6)

has to be small, just as required for an ordinary OPO. The condition \( \Delta k L \approx \pi \) defines the parametric gain bandwidth. Because a Bragg grating in a laser medium gives a mode-dependent threshold to the laser, a DFB structure has been useful in forcing single-longitudinal-mode oscillation in a diode laser. However, a typical OPO with a cavity length of a few centimeters has much smaller longitudinal mode spacing than a diode laser. For example, the 1064-nm pumped parametric gain bandwidth in a 1-cm lithium niobate crystal is a few nanometers,\(^{12}\) whereas the free spectral range of a resonator formed in such a crystal is only \( \sim 7.5 \) GHz, or 0.05 nm for 1.5-\( \mu \)m signal wavelength. The densely packed
longitudinal modes near $\Delta k = 0$ make it difficult for an ordinary OPO to oscillate with a single frequency. A DFB OPO will be valuable if a mode-dependent threshold is present near $\Delta k = 0$ and forces the device to oscillate with only one longitudinal mode. Therefore, we impose the phase-matching condition that $\Delta k z = 0$ on Eqs. (4) and verify in the following calculation the presence of a mode-dependent threshold in a DFB OPO. Usually a DFB laser oscillates at a frequency that is detuned slightly from Bragg resonance. From the detuning parameter, $\delta = \beta - \beta_0$, the amount of the detuned frequency is given by $\Delta \omega_0 = \delta L \omega_{se}/n^2 \tau$, where $\omega_{se} r(2\pi = c_0/2(n L)$ is the free spectral range of the signal wave in a Fabry–Perot resonator of length $L$. As can be seen from what follows, $\Delta \omega_0$ of the oscillation modes is only a few free spectral ranges from Bragg resonance and makes a negligible change in the phase-matching condition $\Delta k z = 0$.

Before an OPO reaches oscillation, the pump field stays fairly constant along $z$, or $A_p(z) \sim A_p(0)$. In calculating the oscillation modes of a DFB OPO we can remove Eq. (4d) from the coupled-wave equations under the constant-pump condition. The coupled-wave equations are now reduced to three linear differential equations and can be recast into the matrix form

$$
\begin{align*}
\frac{dR(z)}{dz} &= \begin{bmatrix} -j\delta & -j\bar{k} & -j\bar{k}_s \end{bmatrix} \begin{bmatrix} R(z) \\ S(z) \\ A_i^*(z) \end{bmatrix} \\
\frac{dS(z)}{dz} &= \begin{bmatrix} j\bar{k} & j\delta & 0 \\ j\bar{k}_s & 0 & 0 \end{bmatrix} \begin{bmatrix} R(z) \\ S(z) \\ A_i^*(z) \end{bmatrix} \\
\frac{dA_i^*(z)}{dz} &= \begin{bmatrix} 0 & 0 & \bar{s} \end{bmatrix} \begin{bmatrix} R(z) \\ S(z) \\ A_i^*(z) \end{bmatrix}
\end{align*}
$$

(7)

where the dimensionless quantities $\bar{z} = z/L$, $\bar{\delta} = \delta L$, $\bar{k} = \kappa L$, and $\bar{k}_s, i = \kappa v A_p(0) L$ are introduced into the expression, where $L$ is the length of the DFB nonlinear optical material. Therefore $\bar{z} = 0, 1$ correspond to the locations of the two end facets of the device. Unlike a DFB diode laser, a DFB OPA or OPO has gain only in the phase-matching direction, and the parametric gain results from the pump intensity. The characteristic equation of Eq. (7) is given by

$$
H(D) = D^3 + [\bar{\delta}^2 - (\bar{\Gamma}^2 - \bar{\kappa}^2)]D + j\bar{s} D^2 = 0,
$$

(8)

where $\bar{\Gamma} = \Gamma L = (\bar{k}_s K_j)^{1/2} = (\kappa v A_p(0))^{1/2} A_p(0) L$, where $\Gamma = (\kappa v A_p(0))^{1/2} A_p(0)$ is the gain coefficient defined in an ordinary OPA or OPO. The roots of Eq. (8) have the form $-2j \text{Re}(D_i), jD_i, jD_i^*$, where $D_i$ is a complex number.

To be practical in discussing the device's performance it is useful to find a reasonable range of the parameters, $\bar{k} = \kappa L$, $\bar{\delta} = \delta L$, and $\bar{\Gamma} = (\bar{k}_s K_j)^{1/2}$. In a bulk nonlinear optical material, the evidence of DFB optical parametric oscillation was demonstrated in the mid-infrared wavelength range with a 5-cm-long periodically poled lithium niobate (PPLN) crystal. The photorefractive DFB grating in the PPLN crystal, written by visible light, had a refractive-index change of $\Delta n \sim 10^{-5}$ and a volume of $\approx 100 \times 10^{-3}$ cm$^3$. From the electro-optic effect, a periodic array of microelectrodes atop a bulk nonlinear optical material can also induce a refractive-index change according to the expression $\Delta n = -n r E_{eq} z$, where $E_{eq}$ is the applied electrode field and $r$ is the electro-optic coefficient of the material. Given the largest $r = r_{33} \sim 30 \text{ pm/V}$ in lithium niobate and a moderate electric field $E_{eq} \sim 500 \text{ V/mm}$, an electro-optic DFB grating could provide an index change of $\Delta n \sim 7 \times 10^{-5}$. For sinusoidal index modulation there is a factor of 2 between $\Delta n$ and the $\Delta k$ defined in the complex Fourier series [Eq. (1b)] or, specifically, $\Delta n = 2 \Delta k n$. Therefore the dimensionless DFB coupling coefficient for a photorefractive or electro-optic grating would be in the range $0.5 < \bar{\kappa} < 5$, if $L \sim 5 \text{ cm}$ and $\lambda_0 \sim 3 \text{ mm}$ are assumed. For waveguide devices the microfabrication techniques used for a DFB diode laser can be applied directly to a waveguide-type DFB OPO. However, the length of a waveguide OPO can be a few hundred times that of a DFB diode laser. With typically $\bar{\kappa} \sim 1$ for a $\sim 100$-mm-long DFB diode laser, the dimensionless DFB coupling coefficient can be $\bar{\kappa} \sim 100$ for a centimeter-long waveguide DFB OPO. As was shown above, frequency detuning $\bar{\delta} = \delta L = (\beta - \beta_0) L = \pi \Delta \omega_0/\omega_{se}$ is the amount of the signal-frequency shift normalized to the free spectral range multiplied by $\pi$. Therefore the maximum value of $|\bar{\delta}|$ can be $\sim 30$ for the resonance modes confined to the central 10% parametric bandwidth, if the parametric bandwidth is the amount of the free spectral range. To estimate a reasonable value for the dimensionless parametric gain coefficient, $\bar{\Gamma} = L(\omega_{se} \omega_{id} \omega_{id}^2) [A_i(0)/2] / (n p s c_0^2)^{1/2}$, we first assume a signal wavelength that is approximately twice the idler wavelength. For example, a 1064-nm-pumped congruent PPLN OPO at 100 °C, producing wavelengths $\lambda_s = 3393$ and $\lambda_i = 1550$ nm in vacuum, has the material parameters, $n_p = 2.16$, $n_s = 2.09$, $n_i = 2.14$, and $d \sim 15 \text{ pm/V}$. For a 1-W continuous-wave pump laser focused to an effective waist radius of 80 µm in a 5-cm bulk PPLN crystal, dimensionless gain coefficient $\bar{\Gamma} \sim 0.2$. If 1-W pump power is focused into a 10-µm-diameter PPLN waveguide of the same length, the dimensionless gain coefficient becomes $\bar{\Gamma} \sim 3$. For pulsed pump lasers producing more than kilowatts of power, $\bar{\Gamma}$ is increased to a few tens in a bulk nonlinear optical material. In what follows, we solve the coupled-wave equations and discuss the device’s performance within the above parameter ranges.

### 3. DISTRIBUTED-FEEDBACK OPTICAL PARAMETRIC AMPLIFIER

A laser amplifier provides optical gain to a seeding laser. In a DFB OPA, the idler is the preferred seeding wave, as its wavelength is not in resonance with the Bragg grating. It is possible to solve Eq. (7) analytically at zero tuning $\bar{\delta} = 0$ and to compare the solutions with known properties of an ordinary OPA and a DFB structure. At zero detuning, one finds that $\text{Re}(D_i) = 0$ and the three eigenvalues from the characteristic equation (8) are $0$, $\bar{\delta} \pm \bar{\Gamma}^{1/2}$. It can be seen that DFB parametric gain coefficient $\bar{\Gamma}$ is enhanced by ordinary parametric gain $\bar{\Gamma}$ through DFB coupling coefficient $\bar{\kappa}$. However, the performance of such a device is subject to specific boundary conditions. For example, one advantage of such a device compared with a conventional DFB laser is its ability to function as an OPA and a difference-frequencey generator (DFG). In a typical DFB laser amplifier it is difficult to inject a seed signal into the DFB structure with the seed-laser wavelength near Bragg resonance.
reflection loss and unwanted optical feedback often occur to the seeding laser in a typical DFB laser amplifier. When a DFB OPA is seeded with the idler wave, however, one obtains a signal wave from difference frequency generation and an amplified idler wave from optical parametric amplification without the need to consider reflection loss or unwanted optical feedback. For boundary conditions $R(0) = 0$, $S(0) = 0$, and $A_i(0)$, the solutions to Eq. (7) at zero detuning are

$$R(\bar{z}) = \frac{\bar{g}}{j \kappa_s^*} \frac{\sinh(\bar{g} \bar{z})}{1 + (\kappa^2 / \bar{g}^2) \cosh(\bar{g} \bar{z})} A_i(0),$$  

(9a)

$$S(\bar{z}) = \frac{\kappa}{\kappa_s^*} \frac{\cosh(\bar{g} \bar{z}) - \cosh(\bar{g})}{1 + (\kappa^2 / \bar{g}^2) \cosh(\bar{g})} A_i(0),$$  

(9b)

$$A_i(\bar{z}) = \frac{(\kappa^2 / \bar{g}^2) \cosh(\bar{g}) + \cosh(\bar{g} \bar{z})}{1 + (\kappa^2 / \bar{g}^2) \cosh(\bar{g})} A_i(0).$$  

(9c)

Without the DFB structure or $\kappa = 0$, Eq. (9b) disappears and Eqs. (9a) and (9c) are reduced to the signal and idler expressions for an ordinary OPA. With some algebraic effort, one can verify the conservation of photon numbers:

$$\frac{I_f(1) + I_b(0)}{\omega_i} = \frac{I_f(1) - I_i(0)}{\omega_i}$$  

(10)

at the outputs of this device. Because Eqs. (9) do not contain any poles, this device cannot function as an oscillator at $\delta = 0$ for the given boundary conditions. Figure 2 illustrates the growth of the forward signal, backward signal, and idler intensities for $\bar{\Gamma} = 0.5$, $\omega_i / \omega_s = 2.2$, $\bar{k} = 0.5$, and $n_i / n_s = 1$. In the figure the intensity gains for the forward signal, the backward signal, and the idler waves are defined to be $\eta_f(\bar{z}) = I_f(\bar{z}) / I_f(0)$, $\eta_b(\bar{z}) = I_b(\bar{z}) / I_b(0)$, and $\eta_i(\bar{z}) = [I_i(\bar{z}) / I_i(0)] - 1$, respectively. The gain value $\bar{\Gamma} = 0.5$ in the figure falls into the range of

$$R(\bar{z}) = R(0) \cosh(\bar{g} \bar{z}) + \frac{\bar{g}}{j \kappa_s^*} A_i(0) \sinh(\bar{g} \bar{z}),$$  

(12a)

$$S(\bar{z}) = \frac{\bar{k}}{\bar{g}} R(0) \sinh(\bar{g} \bar{z}) + \frac{\bar{k}}{\kappa_s^*} A_i(0) \cosh(\bar{g} \bar{z}),$$  

(12b)

$$A_i(\bar{z}) = \frac{\bar{k}}{\bar{g}} R(0) \sinh(\bar{g} \bar{z}) + A_i(0) \cosh(\bar{g} \bar{z}).$$  

(12c)
For \( \kappa = 0 \) this set of solutions is also reduced to that of an ordinary OPA, subject to boundary conditions \( R(0) \) and \( A_\delta^*(0) \). However, the expressions in Eqs. (12) take full advantage of the enhanced gain coefficient \( \tilde{g} = (\kappa^2 + \Gamma^2)^{1/2} \) in such a DFB structure. In the high-gain regime, the amplitudes of all three waves enjoy exponential growth in the \( +z \) direction with DFB-enhanced gain coefficient \( \tilde{g} \). Whether a solution is possible depends on whether the boundary conditions are physically reasonable. Because vacuum noise contains all possible initial photons over the whole spectrum, the boundary conditions for obtaining Eqs. (12), in particular \( S(0) = \kappa A_\delta^*(0)/\kappa_\delta^* \), could be statistically possible in a high-gain, single-pass optical parametric generation (OPG) process. In such an OPG process, both the forward signal and the idler waves grow exponentially along \( +z \), and, because of instantaneous backscattering of the forward signal wave from the local DFB structure, the backward signal wave also grows along \( +z \). Further experimental studies are necessary to verify the peculiar solutions [Eqs. (12)].

Previously we focused on device performance at zero detuning. The analytic solutions at Bragg resonance \( \delta = 0 \) have been useful for comparison of the theoretical model against some known properties of an ordinary OPA and a DFB structure. However, for some detuning value \( \delta \) we show in what follows that the OPA and DFG gain is drastically enhanced near some mode resonances. By using numerical techniques (refer to Appendix A) we can solve Eq. (7) to obtain

\[
\begin{bmatrix}
R(1) \\
S(1) \\
A_\delta^*(1)
\end{bmatrix} = \mathbf{B}(1) \begin{bmatrix}
R(0) \\
S(0) \\
A_\delta^*(0)
\end{bmatrix},
\]

\[
\mathbf{B}(1) = \begin{bmatrix}
b_{11}(1) & b_{12}(1) & b_{13}(1) \\
b_{21}(1) & b_{22}(1) & b_{23}(1) \\
b_{31}(1) & b_{32}(1) & b_{33}(1)
\end{bmatrix}.
\]

(13)

Under the OPA boundary conditions, i.e., \( R(0) = 0, S(0) = 0, \) and \( A_\delta^*(0) \), the output signal and the idler are given by

\[
R(1) = \frac{b_{12}(1) b_{13}(1)}{b_{22}(1) b_{23}(1)} A_\delta^*(0),
\]

\[
S(0) = \frac{b_{22}(1) A_\delta^*(0)}{b_{23}(1)},
\]

\[
A_\delta^*(1) = \frac{b_{23}(1) A_\delta^*(0)}{b_{33}(1)}.
\]

(14a, 14b, 14c)

Figure 3 is a contour plot of OPA intensity amplification ratio \( I_\delta(1)/I_\delta(0) \), or \( \eta(1) + 1 \), in the \( \Gamma - \delta \) plane for DFB coupling coefficient \( \kappa = 1 \). The contours show the gain and detuning values, \( \Gamma \) and \( \delta \), that correspond to the idler amplification ratios \( I_\delta(1)/I_\delta(0) = \cosh^2(1), \cosh^2(2), \ldots \cosh^2(8) \). Because the amplification ratio for an ordinary OPA is given by \( I_\delta(1)/I_\delta(0) = \cosh^2 \Gamma \) for the given boundary conditions, \( \eta(1) + 1 \) the contour curves relative to the horizontal dotted lines \( \Gamma = 1, 2, \ldots, 8 \) give a qualitative gain comparison of a DFB OPA and an ordinary OPA. It is evident from the figure that a DFB OPA operating at \( \delta = 0 \) and large \( \Gamma \) has less gain than an ordinary OPA. However, near some detuning values the OPA gain becomes significant and the device starts to oscillate for some threshold gain \( \Gamma \) at which OPA amplification ratio \( I_\delta(1)/I_\delta(0) \) becomes unbounded. In Fig. 3 six resonance modes are centered in the eyelike circles and are symmetric with respect to \( \delta = 0 \). The location of a resonant mode gives the threshold gain and detuning values that cause \( |b_{22}(1)| = 0 \) in Eqs. (14). The frequency detuning between the lowest-threshold modes is the bandgap of the DFB structure. It should be pointed out that Eqs. (14) do not take into account any end reflections of the mixing waves from the device boundaries at \( \tilde{z} = 0 \) and \( \tilde{z} = 1 \). In what follows, we first study the basic mode properties of a DFB OPO without end reflections and then consider the reflections to show various ways of mode selection and threshold adjustment.

4. DISTRIBUTED-FEEDBACK OPTICAL PARAMETRIC OSCILLATOR

From Eq. (13), the forward transmittance, backward transmittance, forward reflectance, and backward reflectance of the signal waves are

\[
\frac{|R(1)|^2}{|R(0)|} = \frac{b_{11}(1)b_{22}(1) - b_{21}(1)b_{12}(1)}{b_{22}(1)},
\]

\[
S(1) = 0, \quad A_\delta^*(0) = 0, \quad \Gamma = 0, \quad \delta = 0, \quad \delta = 0.
\]

(15a, 15b, 15c)
\[ \frac{R(1)}{S(1)} = \frac{b_{12}(1)^2}{b_{22}(1)}, \quad R(0) = 0, \quad A \xi(0) = 0, \]

(15d)

respectively. At resonance, Eqs. (15) become infinite and the device produces a finite output without an input. Accordingly, the steady-state resonance for a DFB OPO occurs at \( b_{22}(1) = 0 \), as given above. The parametric gain and detuning values that make \( b_{22}(1) = 0 \) define the threshold gain and the oscillation frequency of a resonant mode. Matrix element \( b_{22}(1) \) can be expressed explicitly by the inverse Laplace transform

\[ b_{22}(\tilde{z}) = \mathcal{L}^{-1} \left[ \frac{s(s + j\bar{\delta}) - \bar{\Gamma}^2}{s^3 + \left( \bar{\delta}^2 - (\bar{\kappa}^2 + \bar{\Gamma}^2) \right)s + j\bar{\delta}^2} \right], \]

(16)
evaluated at \( \tilde{z} = 1 \). According to Eq. (16), the mode location is independent of the relative phase between the parametric coupling coefficients \( \bar{\kappa} \) and \( \bar{\kappa}_s \) or is irrelevant to the initial pump phase. By solving \( b_{22}(1) = 0 \) we show in Fig. 4 the first three longitudinal-mode branches, labeled \( m = 1, 2, 3 \), in the plane of threshold gain \( \Gamma_{\text{th}} \) and frequency detuning \( \bar{\delta} \) for various DFB coupling coefficients \( \bar{\kappa} \). This plot is valid for a DFB OPO without facet reflections. Each point on a solid curve of Fig. 4 designates a frequency detuning value and threshold gain of an OPO longitudinal mode at a given \( \bar{\kappa} \) value. DFB coupling coefficient \( \bar{\kappa} \) varies continuously along the solid curves, as indicated by the numbers marked on the dashed lines. For example, with DFB coupling coefficient \( \bar{\kappa} = 0.1 \), the first OPO mode starts to oscillate with threshold gain \( \Gamma_{\text{th}} = 10 \) and frequency detuning \( \bar{\delta} = 2.49 \). The mode branches are symmetric with respect to the zero detuning line \( \bar{\delta} = 0 \), and only those with positive frequency detuning are shown in the figure. As expected, the larger the DFB coupling coefficient, the lower the threshold gain. Like a DFB laser, a DFB OPO has mode-dependent thresholds that often result in the crucial consequence of single-mode oscillation in a DFB laser. The threshold gain for a symmetric singly resonant OPO (SRO) is given by \( R \cosh \Gamma_{\text{th}} = 1 \),\(^{18} \) where \( R \) is the reflectance of the resonator mirrors at the signal wavelength. For comparison, we plot in the same figure by two dashed--dotted lines the threshold gains of two SROs with \( R = 12.6\% \), 95\%. The value \( R = 12.6\% \) equals the optical reflectance from an uncoated, polished lithium niobate crystal with \( n = 2.1 \). To have the same threshold gain as that for an \( R = 12.6\% \) (95\%) SRO, a DFB OPO with no end reflections requires a coupling coefficient of \( \bar{\kappa} = 2 \) (20) to oscillate at the \( m = \pm 1 \) mode.

The width of the stop band or the bandgap of the DFB OPO is the horizontal distance between the two symmetric lowest-threshold modes in Fig. 4. All resonances can occur only outside the stop band. In the low-gain limit, the width of the stop band can be estimated from the characteristic equation (8). Given \( \Gamma \sim 0 \), the three eigenvalues are \( D \sim 0, \pm \sqrt{\kappa^2 - \delta^2} \). According to Eq. (2c), the propagation constant of the modes is given by \( \beta_0 + \text{Im}(D) \), where \( \text{Im}(D) \) is the imaginary part of eigenvalue \( D \). Eigenvalue \( D = 0 \) does not attenuate the wave propagation. To ensure propagation modes, the other two eigenvalues must be imaginary numbers or \( |\delta| > \bar{\kappa} \). Therefore the stop-band width in the low-gain limit is approximately equal to \( 2\bar{\kappa} \). This result is consistent with that for a DFB laser and is clearly shown in Fig. 4.

In addition to its simplicity, the most important feature of a DFB laser is that the laser thresholds for different longitudinal modes are different. In a homogeneously gain broadened laser medium, the lowest threshold mode oscillates first and becomes the dominant oscillation mode of the laser at the steady state. This mode-dependent threshold gain can also be found from a DFB OPO. Figure 5 shows the mode threshold gain versus the DFB coupling coefficient for the first six modes \((m = \pm 1, \pm 2, \pm 3)\) in a DFB OPO without end reflections. In the figure, the mode influence on gain is not obvious for \( \bar{\kappa} \sim 0 \), but it becomes evident when \( \bar{\kappa} \) deviates only slightly from the zero value. For example, at \( \bar{\kappa} = 5 \) the threshold gain of the first modes \((m = \pm 1)\) is lower than that of the second modes \((m = \pm 2)\) by \( \Delta \Gamma_{\text{th}} = 2.38 \), or \( \sim 20 \) dB, which is large enough for mode selection in an optical oscillator. Although in this calculation the structure’s symmetry gives symmetric threshold gain for positively and negatively detuned modes, the 

\[ \Delta \Gamma_{\text{th}} = 2.38 \]

Fig. 4. Threshold gain and frequency detuning of the first three longitudinal-mode branches \((m = 1, 2, 3)\) of a DFB OPO without facet reflections. Each point on a solid curve designates a frequency detuning value and threshold gain of a longitudinal mode at a given \( \bar{\kappa} \). DFB coupling coefficient \( \bar{\kappa} \) varies continuously along the solid curves, as indicated by the values on the dashed lines. For comparison, the threshold gains of conventional symmetric singly resonant OPOs (SROs) with mirror reflectances of 12.6\% (95\%) are shown by dashed–dotted lines.

Fig. 5. Threshold gain of a DFB OPO versus DFB coupling coefficient \( \bar{\kappa} \) for the first six modes. Mode-dependent threshold gain is evident for nonzero \( \bar{\kappa} \).
negatively detuned mode branches, slight asymmetry, for instance in facet reflections or structure design, can remove the threshold degeneracy. This symmetry breaking to threshold gain will become evident when we incorporate end reflections into the calculation.

It should be pointed out that the concept of homogeneous gain broadening has been historically associated with a laser. To the best of our knowledge, the gain-broadening mechanism associated with nonlinear polarizations has never been discussed or cited. Recently we observed a homogeneous gain-broadening phenomenon from a PPLN OPA seeded by a megahertz linewidth diode laser.21 Even though the pump intensity was increased to a level strong enough to produce OPG, the strongly pumped OPA still produced a transform-limited linewidth. The usual broad OPG spectrum was homogeneously narrowed and quenched to the seed-laser linewidth. The usual broad OPG spectrum was homogeneously narrowed and quenched to the seed-laser linewidth. This observation, along with the mode-dependent threshold gain in Fig. 5, fully supports the assertion of single-longitudinal-mode operation of a DFB OPO.

In an ordinary DFB laser the mode locations are sensitive to end reflections, including the reflectance and the reflection phase. To investigate this effect in a DFB OPO we impose the following boundary conditions on steady-state resonance:

\[
R(0) = \sqrt{R_1} \exp(j \phi_1) S(0), \quad \bar{z} = 0, \quad (17a)
\]

\[
S(1) = \sqrt{R_2} \exp(j \phi_2) R(1), \quad \bar{z} = 1, \quad (17b)
\]

where \( R_{1,2} \) and \( \phi_{1,2} \) are the reflectances and the reflection phases, respectively, at the first and second OPO facets. Assuming that \( A_i^+(0) = 0 \) for a DFB SRO with negligible end reflections at the idler wavelength and substituting boundary conditions (17) into Eq. (13), one obtains the steady-state condition at mode resonance:

\[
\left| \frac{b_{22}(1) \sqrt{R_1} \exp(j \phi_1) + b_{22}(1)}{b_{12}(1) \sqrt{R_2} \exp(j \phi_2)} - \sqrt{R_2} \exp(j \phi_2) \right| = 0.
\]

(18)

For zero reflections, \( R_{1,2} = 0 \), Eq. (18) is reduced to the previous resonance condition, \( |b_{22}(1)| = 0 \). In a DFB diode laser the end reflections have two effects, mode shift and threshold adjustment. To illustrate this property for a DFB OPO, we plot in Fig. 6 the mode loci with variable phase \( \phi_1 \) in the \( \Gamma_{th} - \delta \) plane, assuming that \( R_2 = 0 \) and \( \bar{\kappa} = 2 \). The continuous and dashed curves correspond to the reflectances of \( R_1 = 12.6\% \) and \( R_1 = 99\% \), respectively, at the upstream OPO facet. A resonance mode moves along the curves when \( \phi_1 \) is varied and settles at the location of the next longitudinal mode after a 2π phase change in \( \phi_1 \). The mode locations for \( \phi_1 = 0, \pi/2, \pi \) are indicated by circles, triangles, and rectangles, respectively. Usually a symmetric DFB laser has a bandgap near Bragg resonance or zero detuning, \( \delta = 0 \). A DFB OPO is no exception, as can be seen from Figs. 3 and 4. However, choosing reflection phase \( \phi_1 = \pi/2 \) results in the appearance of a gap mode at Bragg resonance in Fig. 6. Facet reflection \( R_1 \) permits the adjustment of the threshold gain of the gap mode. In Fig. 6 a high-threshold gap mode becomes a low-threshold gap mode when the first facet reflectance is increased from \( R_1 = 12.6\% \) to \( R_1 = 99\% \). The condition for creating such a low-threshold gap mode in a DFB OPO turns out to be the same as that in an ordinary DFB laser.20 Therefore a DFB OPO with uniform sinusoidal dielectric modulation has mode-shift and threshold-adjustment properties similar to those of an ordinary DFB laser. Compared to a DFB diode laser, a DFB OPO has additional optical waves, namely, idler and pump waves, in the source medium. As is shown in Section 5 below, it is possible to manipulate, for example, the idler phase in a cascaded DFB OPO and control its mode thresholds and frequencies.

5. TAPERED DISTRIBUTED-FEEDBACK OPTICAL PARAMETRIC AMPLIFIER AND OSCILLATOR

The Bragg grating in a DFB OPA–OPO provides spectrally selective optical feedbacks to the mixing waves. In the calculation above, the parameter that contains the spectral information is detuning parameter \( \bar{\delta} \), and that which contains the feedback strength is DFB coupling coefficient \( \bar{\kappa} \). It is advantageous to have more parameter freedom for optimizing the gain and spectral performance of a DFB OPA or OPO. One can obtain additional parameter freedom by tapering a DFB structure in which the DFB parameters \( \bar{\delta} \) and \( \bar{\kappa} \) are functions of distance. The matrix formulation developed above can be readily extended to calculating a cascaded DFB OPA–OPO structure with different component sections defined by different sets of structure parameters. In the calculation, arbitrary optical phase shifts between adjacent DFB sections, which result from structure defects or intentional fabrications, can be inserted between the matrices that represent individual DFB sections. In what follows, we first derive the expression that governs a cascaded DFB
OPA–OPO and use the expression to study the mode behavior of a two-section DFB OPO with variable signal and idler phases between the two DFB sections.

From Eqs. (2) and (13), the complex amplitudes of the signal and idler waves at input $\bar{z}=0$ and at arbitrary position $\bar{z}$ of a uniform DFB OPA–OPO are related by

$$
\begin{bmatrix}
E_f(\bar{z}) \\
E_b(\bar{z}) \\
E_i^*(\bar{z})
\end{bmatrix} =
\begin{bmatrix}
\exp(-j\bar{\beta}_0 \bar{z}) & 0 & 0 \\
0 & \exp(j\bar{\beta}_0 \bar{z}) & 0 \\
0 & 0 & \exp(j\bar{k}_i \bar{z})
\end{bmatrix}
\times
\begin{bmatrix}
E_f(0) \\
E_b(0) \\
E_i^*(0)
\end{bmatrix},
$$

where the dimensionless quantities $\bar{\beta}_0 = \beta_0 \lambda$ and $\bar{k}_i = k_i \lambda$ are introduced. Suppose that a DFB OPA–OPO of length $L$ is divided into $N$ sections, with the $q$th section, of length $\bar{z}_q$, having DFB parameters $\bar{k}_q$, $\Gamma_q$, and $\bar{\delta}_q$. With the specified DFB parameters, the $q$th DFB section is characterized by a unique transfer matrix $\tilde{B}_q(\bar{z}_q)$ that relates the input and output field envelopes according to Eq. (A5) below, provided that the DFB section contains many DFB grating periods and that the Fourier series, Eq. (1), is valid. This assumption is usually true, because the DFB grating period in the infrared or visible spectrum is of the order of a micrometer and a 1-mm section of such a device contains approximately $10^3$ periods. Furthermore, suppose that a signal phase and an idler phase, $\phi_{s,q}$ and $\phi_{i,q}$, respectively, are introduced between the $q$ and the $q+1$ sections as a result of structural defects or intentional implementations. The complex amplitudes of the signal and idler waves between the input and output of the $N$-section cascaded OPA–OPO are therefore related by

$$
\begin{bmatrix}
E_f(1) \\
E_b(1) \\
E_i^*(1)
\end{bmatrix} =
\prod_{q=1}^{N}
\begin{bmatrix}
\exp(-j\phi_{s,q}) & 0 & 0 \\
0 & \exp(j\phi_{s,q}) & 0 \\
0 & 0 & \exp(j\phi_{i,q})
\end{bmatrix}
\times
\begin{bmatrix}
E_f(0) \\
E_b(0) \\
E_i^*(0)
\end{bmatrix},
$$

where $\phi_{s,q} = \bar{\beta}_0 \bar{z}_q + \phi_{s,q}$, $\phi_{i,q} = \bar{k}_i \bar{z}_q + \phi_{i,q}$, and $\bar{\gamma}_q = \bar{z}_q = 1$.

For a symmetric cascaded structure containing two identical DFB sections, Eq. (20) is reduced to

$$
\begin{bmatrix}
E_f(1) \\
E_b(1) \\
E_i^*(1)
\end{bmatrix} =
\begin{bmatrix}
\exp(-j\bar{\beta}_0 \bar{z}/2) & 0 & 0 \\
0 & \exp(j\bar{\beta}_0 \bar{z}/2) & 0 \\
0 & 0 & \exp(j\bar{k}_i \bar{z}/2)
\end{bmatrix}
\times
\begin{bmatrix}
E_f(0) \\
E_b(0) \\
E_i^*(0)
\end{bmatrix},
$$

or, in terms of the field envelopes,

$$
\begin{bmatrix}
R(1) \\
S(1) \\
A_i^*(1)
\end{bmatrix} =
\begin{bmatrix}
\exp(-j\phi_s) & 0 & 0 \\
0 & \exp(j\phi_s) & 0 \\
0 & 0 & \exp(j\phi_i)
\end{bmatrix}
\times
\begin{bmatrix}
R(0) \\
S(0) \\
A_i^*(0)
\end{bmatrix},
$$

wherein the optical phases $\phi_s$ and $\phi_i$ are introduced at the junction of the two DFB sections, and therefore $\phi_s = \bar{\beta}_0 \bar{z}/2 + \phi_{s,q}$ and $\phi_i = \bar{k}_i \bar{z}/2 + \phi_{i,q}$. One can implement signal phase $\phi_s$ by adding or removing a certain length of the DFB period, and the difference between signal and idler phases can be obtained from material dispersion or dynamically controlled by an electric field in an electro-optic material. As both the DFB period and the coherence length between the signal and idler waves are of the order of micrometers for most nonlinear optical materials in the optical spectrum, the parametric gain from the spectrum is of the order of a micrometer and a 1-mm section as a result of structural defects or intentional implementations.

In a DFB laser, a DFB structure with a high-reflection end and an optical phase shift from the end face is equivalent to a structure cascaded from two uniform DFB sections with the same optical phase shift at the center of the structure. For example, a low-threshold gap mode is created in a DFB laser if a quarter-wave phase shift is introduced in both the forward and the backward waves at the middle of a DFB structure without end-face reflections or at a high-reflection end of a uniform DFB structure. As shown in Fig. 6, a gap mode was also created in a DFB OPO with a quarter-wave phase shift introduced at the high-reflection end. This analogy between a DFB diode laser and a DFB OPO is attributable to the boundary condition $A_i^*(0) = 0$ associated with a self-started DFB SRO, which decouples the initial idler amplitude and phase from resonance condition (18) and makes the mode characteristics of a DFB OPO with a high-reflection boundary resemble those of a DFB diode laser given the same boundary condition. A symmetric cascaded DFB OPO, however, is fundamentally different from a symmetric cascaded DFB diode laser because the idler field produced in the first DFB OPO section continuously propagates into the second section. Therefore one would expect that both the signal and the idler phases at the center of a symmetric cascaded DFB OPO would influence the mode characteristics of a cascaded DFB OPO. Using Eq. (23) and keeping $\bar{\kappa} = 2$ and $\phi_s = 0$ yield the tuning the mode frequency and threshold through idler phase $\phi_i$ as shown in Fig. 7(a). As expected from a uniform DFB OPO, the mode locations that correspond to $\phi_s = 0$ are the same as those with $\bar{\kappa} = 2$ in Fig. 4. The stop band is located between the two distorted circles. It can be seen from Fig. 7(a) that varying idler phase $\phi_i$ traces out a mode branch for each mode number $m$ in the $I_{\lambda_{\text{th}}} \bar{\delta}$.
though it is well known that a quarter-wave middle section in a symmetric cascaded DFB OPO in the Fig. 7. Mode loci of a symmetrically cascaded, two-section DFB OPO with that the idler phase is capable of tuning the frequency and threshold of the gap mode in a symmetric cascaded DFB OPO that has a quarter-wave middle section. The idler-phase tuning is unique to a DFB OPO and offers an additional degree of freedom in mode-threshold and mode-frequency control. For more-complicated tapered DFB OPO structures the general expression, Eq. (20), can be used for further analysis.

6. DISCUSSION
In this paper we have developed the theoretical basis of a new class of coherent light source called distributed-feedback optical parametric amplifiers and oscillators. The behavior of this new class of light source constitutes a rich set of physics, which we cannot fully explore with all possible device configurations in a single paper. However, we have discussed the most commonly adopted configuration of this device, in which the pump and idler waves propagate only in the forward direction, the signal wave resonates in the DFB structure, and the mixing waves are plane waves like those in a waveguide. We discuss in what follows possible expansions of the theory, variants of the device configuration, and implementation of the Bragg grating in a nonlinear optical material.

Similarly to a DFB laser, a DFB OPO is most suited for waveguide operation because of its better beam confinement and mature microfabrication technology. Modification of the bulk DFB OPA–OPO theory for a waveguide structure is straightforward by introduction into Eq. (3) of the transversely dependent mode fields and dielectric modulation, given by

\[ \bar{E}_{p,s,i}(x, y, z, t) = \text{Re}[\Phi_{p,s,i}(x, y)E_{p,s,i}(z)\exp(j\omega_{p,s,i}t)], \]

\[ \Delta \varepsilon(x, y, z) = \sum_{i=0}^{N} \Delta \varepsilon_i(x, y)\exp(-jk\vec{g}z), \]

where \( \Phi_{p,s,i}(x, y) \) are the electric field profiles of the transverse waveguide modes of the mixing waves. The net result is the modification of DFB coupling coefficient \( k \) in accordance with the transverse dependence of the waveguide mode and the refractive index. The derivation can be found in most textbooks that describe coupled waves in a dielectric modulated optical waveguide (see, for example, Ref. 22).

For the analysis in this paper, the pump wave was not included because of the constant- or undepleted-pump assumption. For the resonant-mode analysis, which is the primary focus of this paper, this assumption is valid, because the pump is fairly constant before a DFB OPO reaches oscillation. However, to understand the pump-depleted gain of a DFB OPA and the conversion efficiency of a DFB OPO it is necessary to include the pump wave in the analysis. The inclusion of pump-wave equation (4d) generates a set of nonlinear differential equations, and a different numerical technique is required for further investigating the pump-depleted regime of a DFB OPA–OPO.

plane. The mode branches for \( m = \pm 1 \) are even connected to \( m = \pm 2 \) for some idler phase values. The profound influence of the idler phase on mode threshold and frequency results from coherence superposition of the signal fields in the two DFB sections. In each DFB section the signal field is phase matched to the idler field. Therefore the phase reset of the idler field between the DFB sections changes the relative phase of the signal waves in the two DFB sections. Because the resonant mode is a consequence of the longitudinal phase condition imposed on the signal wave, the idler phase controls the signal phase through the parametric process and in turn tunes the mode frequency and threshold of such an oscillator. Figure 7(b) shows the mode loci of a symmetrically cascaded DFB OPO with \( \phi_s = \pi/2 \) and variable \( \phi_i \). In Fig. 7(b), \( \phi_i \) varies with \( \pi/16 \) between adjacent dots, and each mode branch expanded by \( \phi_i \) is clearly shown.
It is fairly common that a nonlinear optical material used for an OPA or an OPO has a small absorption loss at certain wavelengths. Extending coupled-wave theory to accommodate the optical loss is straightforward. With distributed absorption loss, the complex dielectric constant contains a modulated imaginary part together with a real part, given by

\[
e(z) = \varepsilon' + \sum_{i \neq 0} \Delta \varepsilon_i \exp(-jlkg_{z^2}) - j \left[ \varepsilon'' + \sum_{i \neq 0} \Delta \varepsilon'' \exp(-jlkg_{z^2}) \right],
\]

(26)

where \(\varepsilon' - j \varepsilon''\) is the average dielectric constant including the optical loss in \(\varepsilon'\), and \(\Delta \varepsilon_i'\) and \(\Delta \varepsilon_i''\) are the real and imaginary parts of the Fourier amplitudes of the dielectric modulation. With \(\varepsilon'' \ll \varepsilon', \Delta \varepsilon_i' \ll \varepsilon', \) and \(\Delta \varepsilon_i'' \ll \varepsilon''\), refractive index \(n(z)\) and amplitude attenuation coefficient \(a(z)\) seen by an optical wave are given by

\[
n(z) = n + \sum_{i \neq 0} \Delta n_i \exp(-jlkg_{z^2}),
\]

(27)

\[
a(z) = \alpha + \sum_{i \neq 0} \Delta \alpha_i \exp(-jlkg_{z^2}),
\]

(28)

where \(n \approx \sqrt{\varepsilon'}, \Delta n_i \approx \Delta \varepsilon_i'/2n, \alpha = \pi \varepsilon''/(n \lambda v), \Delta n_i \ll n, \Delta \alpha_i \ll \alpha, \) and \(\Delta \alpha_i = \pi \Delta \varepsilon''/(n \lambda v), \) where \(\lambda v\) is the vacuum wavelength of the optical field. Adopting expressions (26)–(28) and substituting Eqs. (2) into the wave equation with optical loss and nonlinear polarization, one obtains the modified coupled-wave equations under the simultaneous phase-matching conditions, \((2 \beta_0 - lk_p)z = 0\) and \((k_p - k_0 - k_i)z = 0\):

\[
\frac{dR(z)}{dz} = -(j \delta + \alpha_s)R(z) - j \kappa_p S(z) - j \kappa_i A_p(z) A_i^*(z),
\]

(29a)

\[
\frac{dS(z)}{dz} = j \kappa_s R(z) + (j \delta + \alpha_s)S(z),
\]

(29b)

\[
\frac{dA_i^*(z)}{dz} = j \kappa_i^* A_p^*(z) R(z) - \alpha_i A_i^*(z),
\]

(29c)

\[
\frac{dA_p(z)}{dz} = -j \kappa_p A_i(z) R(z) - \alpha_p A_p(z),
\]

(29d)

where the DFB coupling coefficient is modified to be \(\kappa_p = 2 \pi \Delta n_i/\lambda_0 = j \Delta \alpha_i\). With the loss terms \(\alpha_{s,i,p}\) in the coupled-wave equations, the general result is the reduction of the parametric gain of a DFB OPA and the increase of the mode threshold of a DFB OPO. To study the detailed behavior of a lossy DFB OPA or OPO, one can start with modified coupled-wave equations (29) and follow the same analysis described in Sections 3–5.

For a more general configuration, the pump wave, and thus the idler wave, may propagate in both longitudinal directions. This configuration could occur in an intracavity-pumped DFB OPO. With all three mixing waves propagating in both longitudinal directions, there are six coupled-wave equations that govern the forward and backward field envelopes of the signal, idler, and pump waves under the phase-matching conditions, \((2 \beta_0 - lk_p)z = 0\) and \((k_p - k_0 - k_i)z = 0\). The fundamental difference between this double-side pumped DFB OPO and the single-side pumped DFB OPO [Eqs. (4)] is in its symmetry along the \(\pm z\) directions. Therefore the mode property of a double-side pumped DFB OPO is expected to be closer to that of a DFB diode laser, except that the parametric gain of an OPO is coupled through nonlinear polarizations but not through electron–hole combinations. A double-side pumped DFB OPO, although it has the same gain-direction symmetry as a DFB diode laser, contains additional optical waves, the pump and idler waves, in the gain medium. The optical phases and intensities of the pump and idler waves will certainly influence the growth of the signal wave and distinguish a double-side pumped DFB OPO from a DFB diode laser.

We leave a detailed study of the double-side pumped DFB OPO for future research.

The forward phase-matching condition, \((k_p - k_0 - k_i)z = 0\), studied in this paper is just one of several possible phase-matching conditions in a DFB OPA or OPO and is the most common condition in practice. In general, given Bragg condition \(2 \beta_0 = lk_p = 0\), there are three sets of coupled-wave equations that correspond to the three possible phase-matching conditions \((k_p - k_0 - k_i)z = 0\), \((k_p + \beta_0 - k_i)z = 0\), and \((k_p - \beta_0 + k_i)z = 0\). Besides forward-phase-matched coupled-wave equations (4), the other two sets of coupled-wave equations, subject to phase-matching conditions \((k_p + \beta_0 - k_i)z = 0\) and \((k_p - \beta_0 + k_i)z = 0\), belong to a class of backward-wave OPA or OPOs. If a quasi-phase-matched nonlinear optical material is used, there is another set of coupled-wave equations that corresponds to the phase-matching condition \((k_p + \beta_0 + k_i - k_{QPM})z = 0\), where \(k_{QPM}\) is the quasi-phase-matching grating vector.

For the three phase-matching conditions \((k_p + \beta_0 - k_i)z = 0\), \((k_p - \beta_0 + k_i)z = 0\), and \((k_p + \beta_0 + k_i - k_{QPM})z = 0\), either or both the signal and the idler waves propagate in the opposite direction from the pump wave. The counterpropagating waves form an internal feedback loop for establishing optical oscillation even without a DFB structure. Our preliminary study shows that, with the DFB structure, the structure feedback loop competes with the internal feedback loop and creates detuned oscillation modes in addition to those at Bragg resonance.

Implementation of the Bragg grating in a nonlinear optical material is crucial for fabricating this useful coherent light source. The first demonstrated DFB OPO utilized a photorefractive grating in a bulk PPLN crystal. For the photorefractive DFB structure, further improvements in the thermal stability and coupling strength of the grating are necessary. Many nonlinear optical materials are also electro-optic materials. The space charge field for producing the photorefractive grating can in some cases be replaced by an external electric field from an array of periodic microelectrodes. As was estimated above, the index modulation amplitude of an electro-optic DFB grating can be of the order of \(\delta n \approx 10^{-4} - 10^{-5}\) in lithium niobate. For a waveguide device, fabricating the
DFB structure can be fairly straightforward by use of the mature microlithographic techniques from the semiconductor industry. For a DFB diode laser a few hundred micrometers in length, a corrugated structure atop the gain medium is sufficient to provide a DFB coupling strength of $\kappa \sim 1.14$. Similarly, the corrugated DFB structure can be integrated with a waveguide DFB OPA or OPO by use of the standard material etching or deposition technique that is widely adopted in semiconductor microfabrications. Compared to a 100-μm long DFB diode laser, a centimeter-long waveguide OPO may have a DFB coupling coefficient a hundred times larger. We indeed anticipate the demonstration of high-performance waveguide DFB OPAs and OPOs in the near future. A DFB OPO has wavelength selectivity in the transparent range of a nonlinear optical material. However, real-time wavelength tunability is always a desirable attribute for an OPO. Some nonlinear optical materials, such as lithium niobate, are also piezoelectric materials. Acousto-optic (AO) index modulation could be a feasible scheme for implementing a real-time variable DFB grating in a DFB OPA or OPO. One can therefore tune the wavelength of an AO DFB OPO by varying the acoustic frequency. In lithium niobate, the refractive-index change in an AO DFB structure can be comparable with that in an electro-optic DFB structure, or $\Delta n \sim 10^{-4} \sim 10^{-5}$, given an acoustic intensity of $\sim 10 \text{ W/cm}^2$ and an AO figure of merit of $6.99 \times 10^{-14} \text{ m}^2/\text{W}$. The ultrasound speed in lithium niobate varies from 3500 to 7000 m/s, depending on the type of acoustic wave and on their propagation direction with respect to the crystal symmetry. The surface acoustic wave (SAW) has the slowest speed and is most promising for forming an AO DFB structure in an optical waveguide. To generate a 1-μm-period first-order AO DFB grating in lithium niobate, the SAW’s frequency is approximately 3–4 GHz and is in the high-frequency limit of the current SAW technology. Like many DFB diode lasers, an AO DFB OPA/OPO can initially adopt a high-order and long-period DFB grating to reduce the operational frequency of the SAW.

7. CONCLUSIONS

In this paper we have developed a coupled-wave theory for distributed-feedback optical parametric amplifiers and oscillators. The performance of forward-phase-matched DFB OPAs and OPOs was investigated in detail. A DFB OPA can have significant parametric gain near mode resonance. When the idler is used as a seeding wave, a DFB OPA does not have the problems of reflection loss and optical feedback associated with an ordinary DFB laser amplifier. From coupled-wave theory we have shown that a DFB OPO retains the basic properties of a DFB diode laser, including mode-dependent threshold gain and mode shift and threshold adjustment from facet reflections. The primary difference between a DFB OPO and a DFB laser is that the former requires phase matching among the mixing waves and the optical gain is direction dependent. Consequently, a forward-phase-matched DFB OPA has the advantage of forward power extraction. This unique property is further revealed in the cascaded version of a DFB OPO, in which the idler phase relative to the signal phase has a profound influence on and control of the oscillation modes. A DFB OPO has the advantages of the wavelength selectivity of an OPO and the mode selectivity of a DFB laser. We expect rich device physics and applications to be discovered in the near future from this new class of coherent light source.

APPENDIX A

The coupled-wave equation, Eq. (7), was solved and cross checked by two independent numerical techniques. The first technique is the so-called eigenvalue–eigenvector technique and the second one employs a Laplace transform. Both techniques avoid numerical integration for differential equations (7).

In the eigenvalue–eigenvector technique, one first solves characteristic equation (8), $H(D) = 0$, to obtain the three roots, $D_1$, $D_2$, and $D_3$. If $D_1$, $D_2$, and $D_3$ are distinct, three linearly independent eigenvectors, $x_1$, $x_2$, and $x_3$, can be obtained from the equation

$$
\begin{bmatrix}
-j \delta & -j \kappa & -j \kappa_s \\
-j \kappa & j \delta & 0 \\
0 & 0 & 0
\end{bmatrix}
x_{1,2,3} = D_{1,2,3} x_{1,2,3}.
$$

If there is a double root or a triple root in the characteristic equation, a more-complicated but standard mathematical procedure is necessary to produce the three linearly independent eigenvectors, $x_1$, $x_2$, and $x_3$. With known $D_1$, $D_2$, and $D_3$ and $x_1$, $x_2$, and $x_3$, the solutions to the signal and idler fields are given by

$$
\begin{bmatrix}
R(z) \\
S(z) \\
A_i(z)
\end{bmatrix} = c_1 x_1 \exp(D_1z) + c_2 x_2 \exp(D_2z) + c_3 x_3 \exp(D_3z),
$$

where the coefficients $c_{1,2,3}$ can be determined from the boundary conditions $R(0)$, $S(1)$, and $A_{0}^{\ast}(0)$. With the column vectors

$$
x_1 = \begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{bmatrix},
\quad x_2 = \begin{bmatrix} x_{12} \\ x_{22} \\ x_{32} \end{bmatrix},
\quad x_3 = \begin{bmatrix} x_{13} \\ x_{23} \\ x_{33} \end{bmatrix},
$$

the $c$ coefficients are given by

$$
\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = M^{-1} \begin{bmatrix} R(0) \\ S(1) \\ A_i^{\ast}(0) \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} \exp(D_1) & x_{22} \exp(D_2) & x_{23} \exp(D_3) \\ x_{31} & x_{32} & x_{33} \end{bmatrix}^{-1} \times \begin{bmatrix} R(0) \\ S(1) \\ A_i^{\ast}(0) \end{bmatrix}.
$$

The resonant condition is equivalent to having unbounded amplitudes of the signal and idler fields for a given set of initial inputs. As a result, the resonance condition is
from which one can obtain threshold gain $\Gamma_{th}$ and frequency detuning $\delta$ of resonant modes.

For the Laplace-transform technique, one takes the Laplace transform of Eq. (7) to obtain

$$
\begin{bmatrix}
  s + j\delta & j\kappa & j\kappa_i \\
  -j\kappa & s - j\delta & 0 \\
  -j\kappa_i & 0 & s
\end{bmatrix}
\begin{bmatrix}
  R(s) \\
  S(s) \\
  A_i^*(s)
\end{bmatrix} =
\begin{bmatrix}
  R(0) \\
  S(0) \\
  A_i^*(0)
\end{bmatrix},
$$

where $s$ is the Laplace transform variable and $R(s)$, $S(s)$, $A_i^*(s)$ are the Laplace transforms of $R(\bar{z})$, $S(\bar{z})$, and $A_i^*(\bar{z})$, respectively. By taking the inverse Laplace transform we can express the signal and output field envelopes at location $\bar{z}$ by

$$
\begin{bmatrix}
  R(\bar{z}) \\
  S(\bar{z}) \\
  A_i^*(\bar{z})
\end{bmatrix} = \mathbf{B}(\bar{z})
\begin{bmatrix}
  R(0) \\
  S(0) \\
  A_i^*(0)
\end{bmatrix},
$$

where transfer matrix $\mathbf{B}(\bar{z})$ is given by

$$
\mathbf{B}(\bar{z}) =
\begin{bmatrix}
  b_{11}(\bar{z}) & b_{12}(\bar{z}) & b_{13}(\bar{z}) \\
  b_{21}(\bar{z}) & b_{22}(\bar{z}) & b_{23}(\bar{z}) \\
  b_{31}(\bar{z}) & b_{32}(\bar{z}) & b_{33}(\bar{z})
\end{bmatrix} = \mathcal{L}^{-1}
\begin{bmatrix}
  \frac{1}{H(s)}
  \begin{bmatrix}
    s(s - j\delta) + \bar{\Gamma}^2 & -j\kappa s & \bar{\kappa} \kappa \\
    j\kappa s & s(s + j\delta) & j\kappa_i(s + j\delta) \\
    -\bar{\kappa} \kappa_i^* & j\kappa_i^*(s + j\delta) & s^2 + \delta^2 - \kappa^2
  \end{bmatrix}
\end{bmatrix},
$$

where $\mathcal{L}^{-1}$ is the inverse Laplace transform operator. $\mathbf{B}(1)$ in Eq. (13) is simply $\mathbf{B}(\bar{z})$ evaluated at $\bar{z} = 1$. The Laplace-transform technique has the advantage of not requiring special treatments for root degeneracy in the characteristic equation. Substituting Eq. (A2) into Eq. (A1) and recasting Eq. (A1) into the form of Eq. (13), one can prove that the resonance conditions $|M| = 0$ and $|b_{22}(1)| = 0$ are equivalent.

ACKNOWLEDGMENTS

This research is supported by the National Science Council of Taiwan under grant NSC 92-2622-L-007-001 and by HC Photonics, Inc., as part of National Tsinghua University project 92A0082J6. The authors appreciate fruitful discussions with J. T. Shy, A. C. Chiang, Y. H. Chen, Y. Y. Lin, F. C. Fan, C. Y. Chien, and K. W. Chang.

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